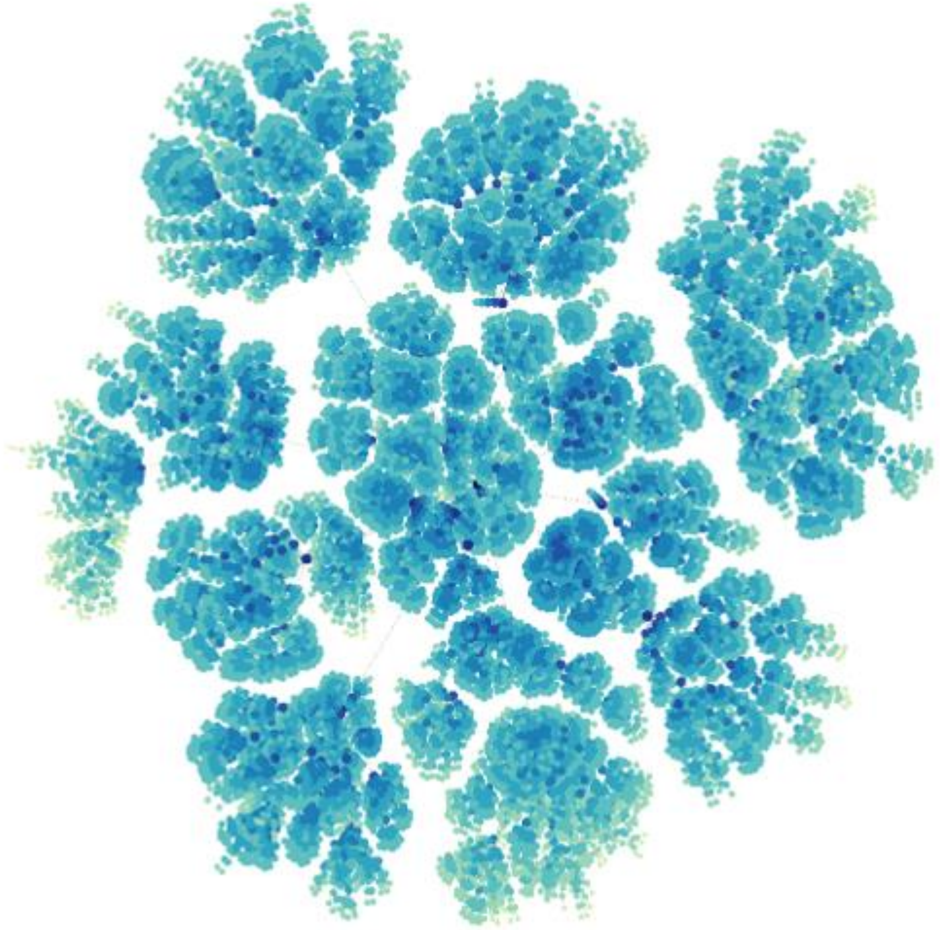
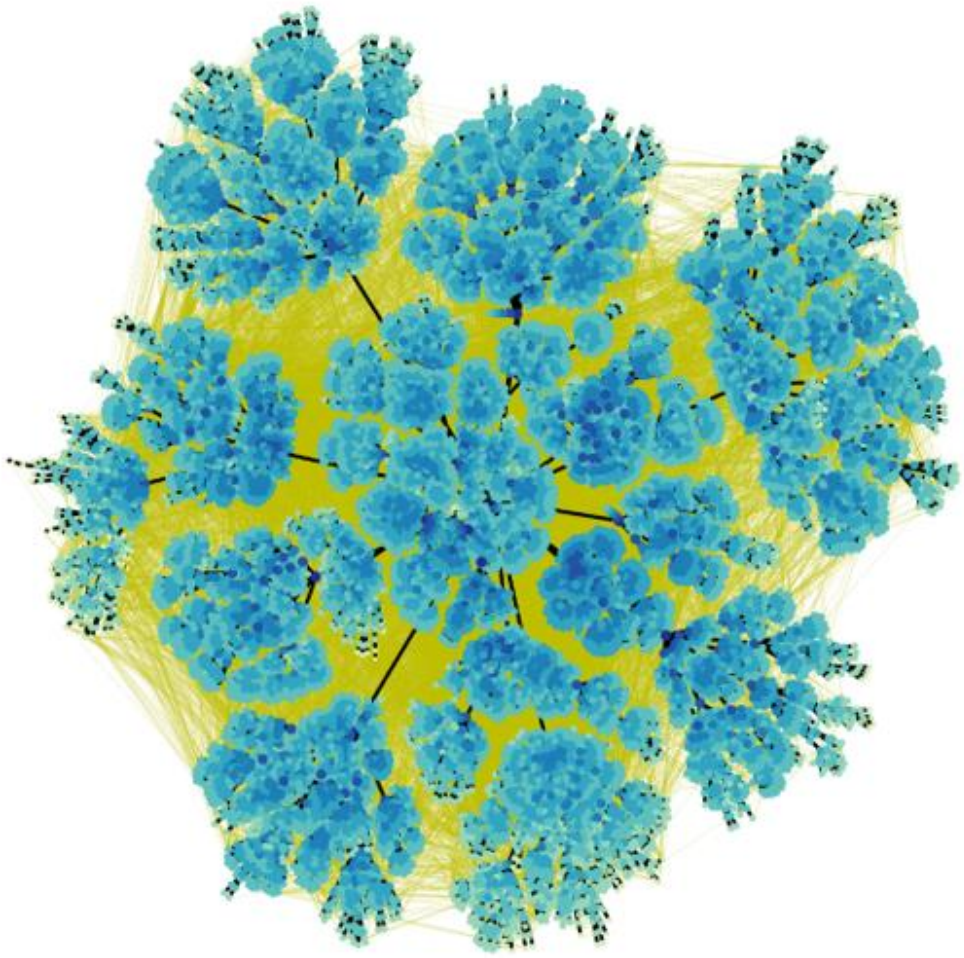
XianFeng Project

Average Shortest Path Distance Minimization via Shortcut Edge Addition problem

Research Background



Organization Tree



Cooperation Network

Significance of Edge Modification

- **Cuts**: removal of existing edges (community partition)

- **Shortcuts**: addition of edges between two nodes

  (argumentation of graph)

- **Small changes on the structure of a graph can have a dramatic effect on its connectivity.**

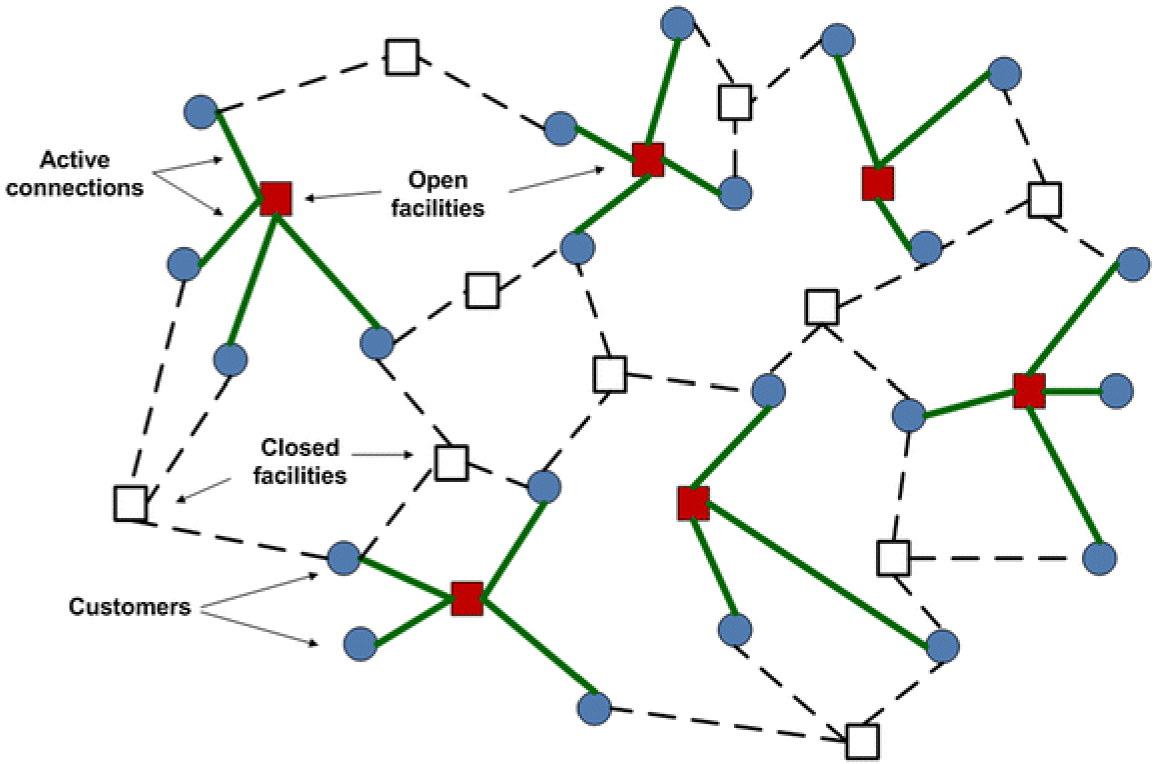
Theoretical perspective：

Facility Location Problem

- Facility Location Problem(选址问题) 是运筹学和计算图形学中的分支。它是一个经典的优化问题，通过选择合适的工厂或仓库地址，在满足客户需要和条件限制的情况下使得成本最少。(NP-hard)

- The k-median Problem. a variant of the ***uncapacitated*** *facility location problem* in graph. (NP-hard)

The k-median Problem



$$
\begin{aligned}
\text{minimize} \quad & \sum_{i=1}^{n}\sum_{j=1}^{m} c_{ij}x_{ij} \\
\text{subject to:} \quad & \sum_{j=1}^{m} x_{ij} = 1 \quad \text{for } i = 1,\cdots,n \\
& \sum_{j=1}^{m} y_j = k \\
& x_{ij} \le y_j \qquad \text{for } i = 1,\cdots,n; j = 1,\cdots,m \\
& x_{ij} \in \{0,1\} \quad \text{for } i = 1,\cdots,n; j = 1,\cdots,m \\
& y_j \in \{0,1\} \quad \text{for } j = 1,\cdots,m \\
& \sum_{i=1}^{n} x_{ij} \le y_j, \text{ for } j = 1,\cdots,m.
\end{aligned}
$$

There exists a polynomial-time *O(1)*-approximation algorithm for k-median Problem with penalties.

Theoretical perspective：

Edge Effectiveness

- Greedy Algorithm
- Heuristic algorithm
- Others : Hierarchical clustering (betweenness)

Experiment

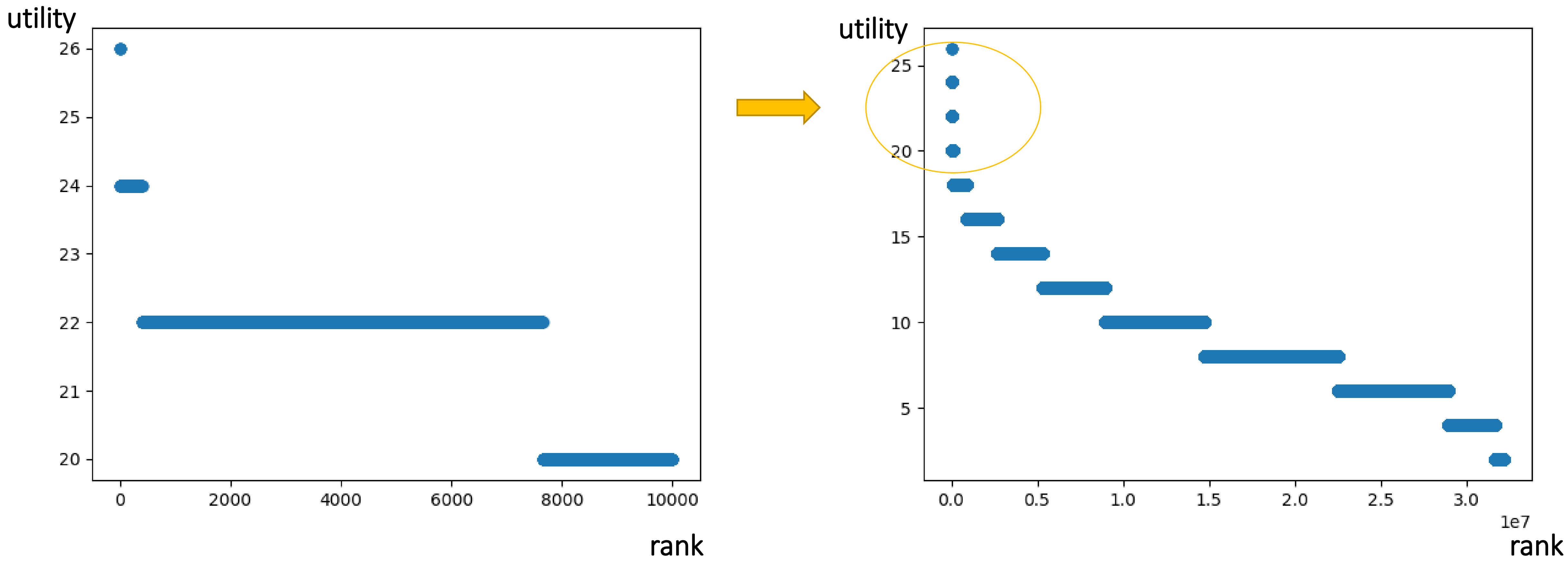
- Path Screening Method （Manos Papagelis. 2015.)
- Better to attach a shortcut on nodes of a shortest path

Input: A Social Graph $G(V, E)$
Output: A Map between Candidate Shortcuts and their utility scores

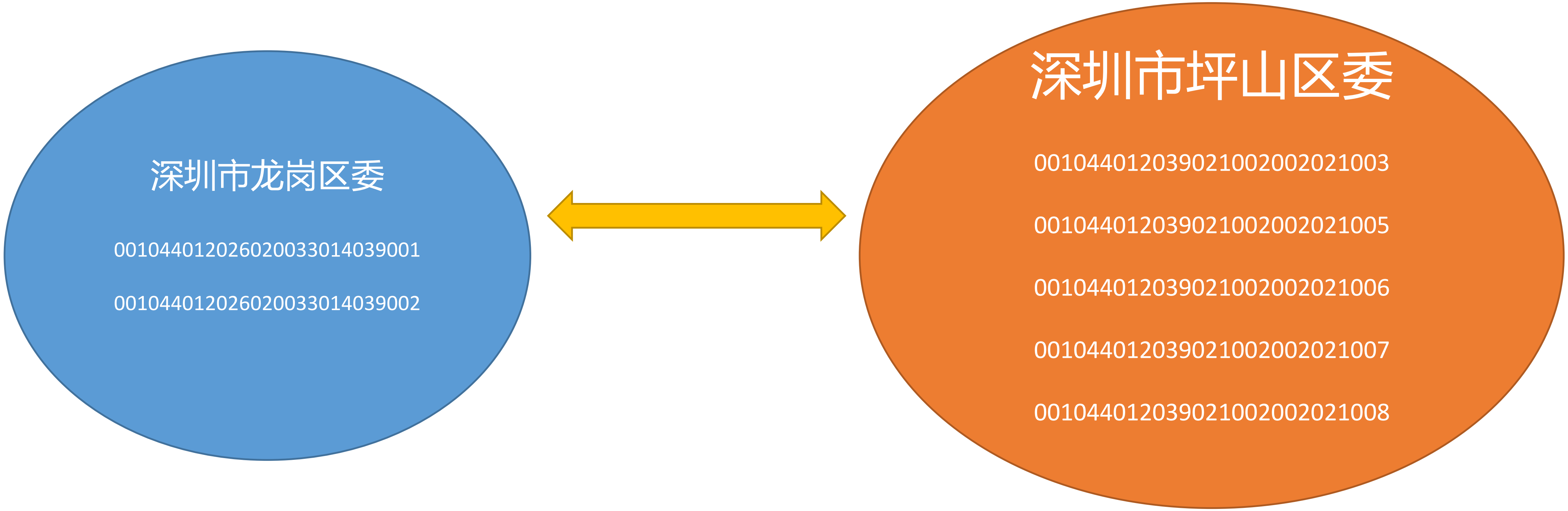
```
U ← ∅;
P = getAllPairsShortestPaths(G);
forall the p ∈ P do
    l = length(p);
    // Variable size of window
    for δ = 2 to l do
        // Slide a δ-size window over nodes of p
        for i = 0 to l − δ do
            x = p[i];                                   // ith node of path p
            y = p[i + δ];                               // (i + δ)th node of path p
            if (x, y) ∉ U then
                u_xy = (δ − 1);
                add < (x, y), u_xy > in U;
            else
                update < (x, y), u_xy + (δ − 1) > in U;
            end
        end
    end
end
return U
```

$$O(n(n\log n+m))+O(n^2\bar{L}(G)\bar{L}(G))$$

Distribution of utility



Best shortcut

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罗湖区
盐田区
大鹏新区(属龙岗区)

Reference

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