

# Explainable Trajectory Representation through Dictionary Learning

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## Introduction

### Background.

- Trajectory representation learning provides great opportunities to understand vehicular traffic patterns.
- It can benefit downstream tasks including trajectory compression, trip time estimation, public transportation route planning.

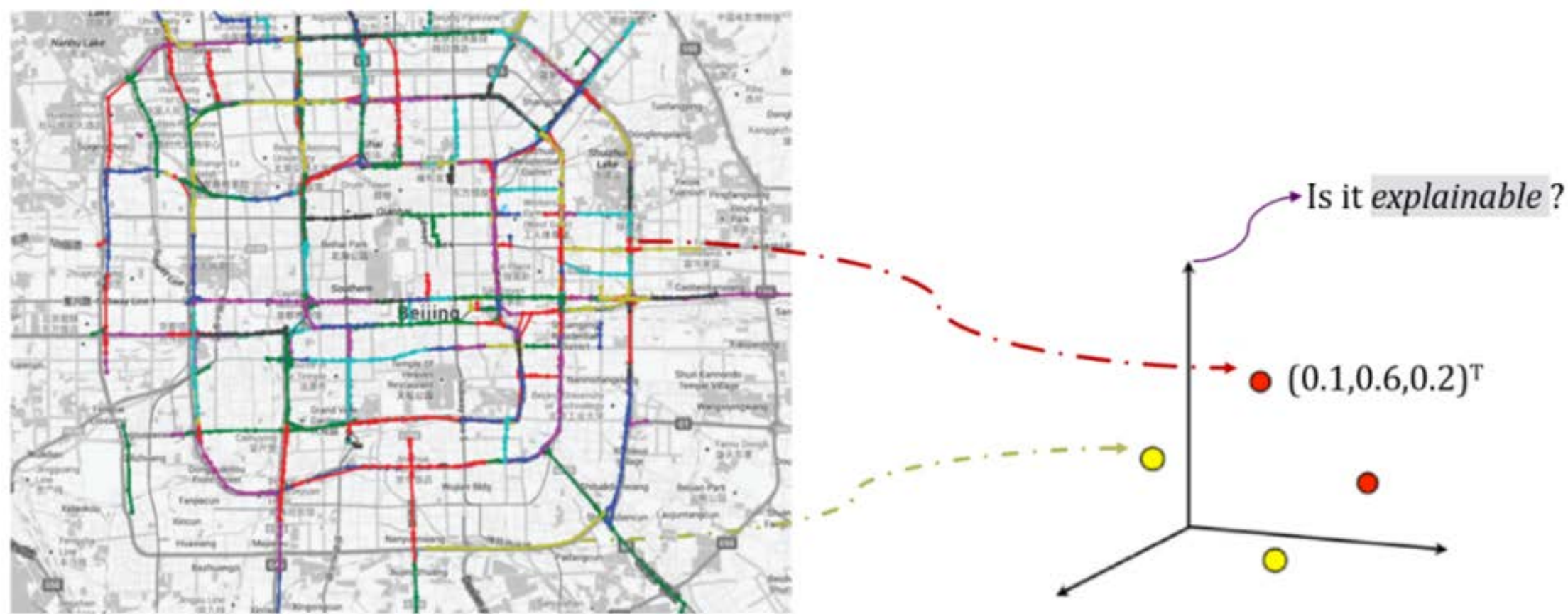


Fig 1. Transforming a trajectory into an embedding vector

### Motivation.

- Embeddings generated by deep-learning methods are usually dense vectors whose dimensions lack semantic meaning.
- It is difficult to interpret the learned representation.

### Purpose.

- Extract common trajectory segments (called pathlet) as a dictionary.
- Represent trajectory by concatenating pathlets from dictionary.
- Generate semantic trajectory representation vectors, each dimension corresponding to a mobility pattern.

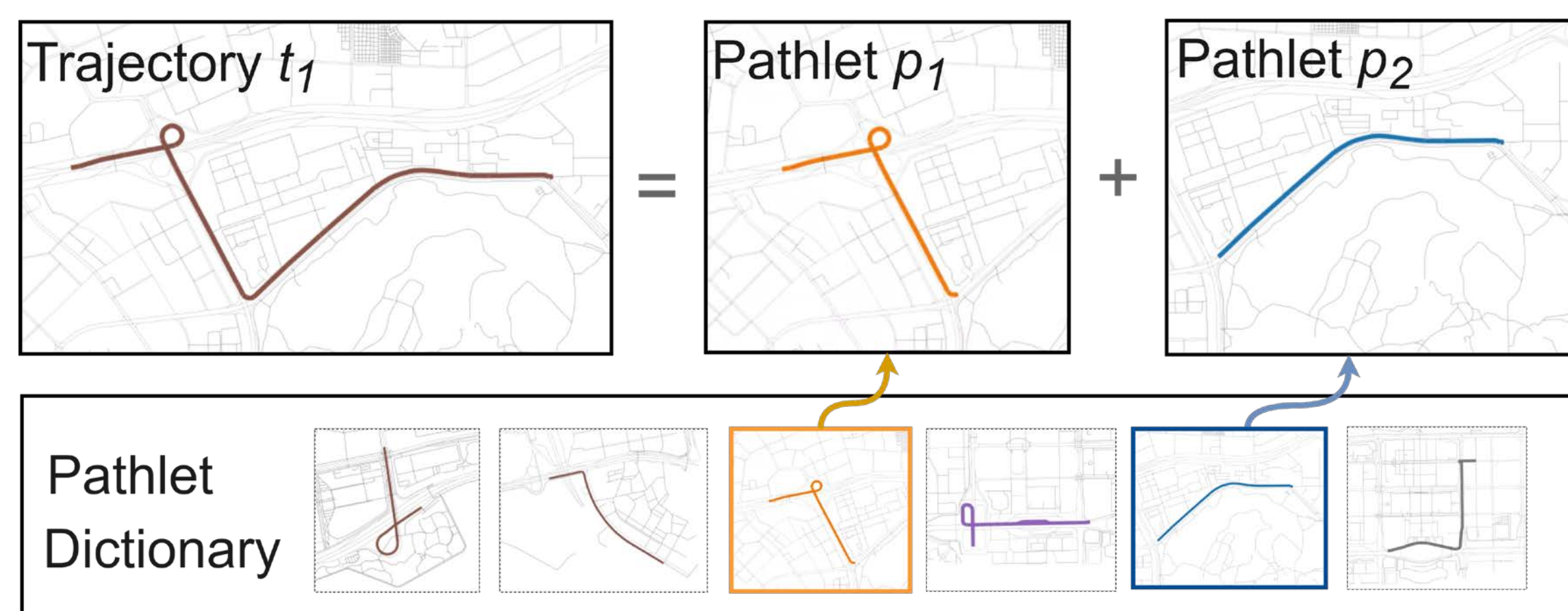
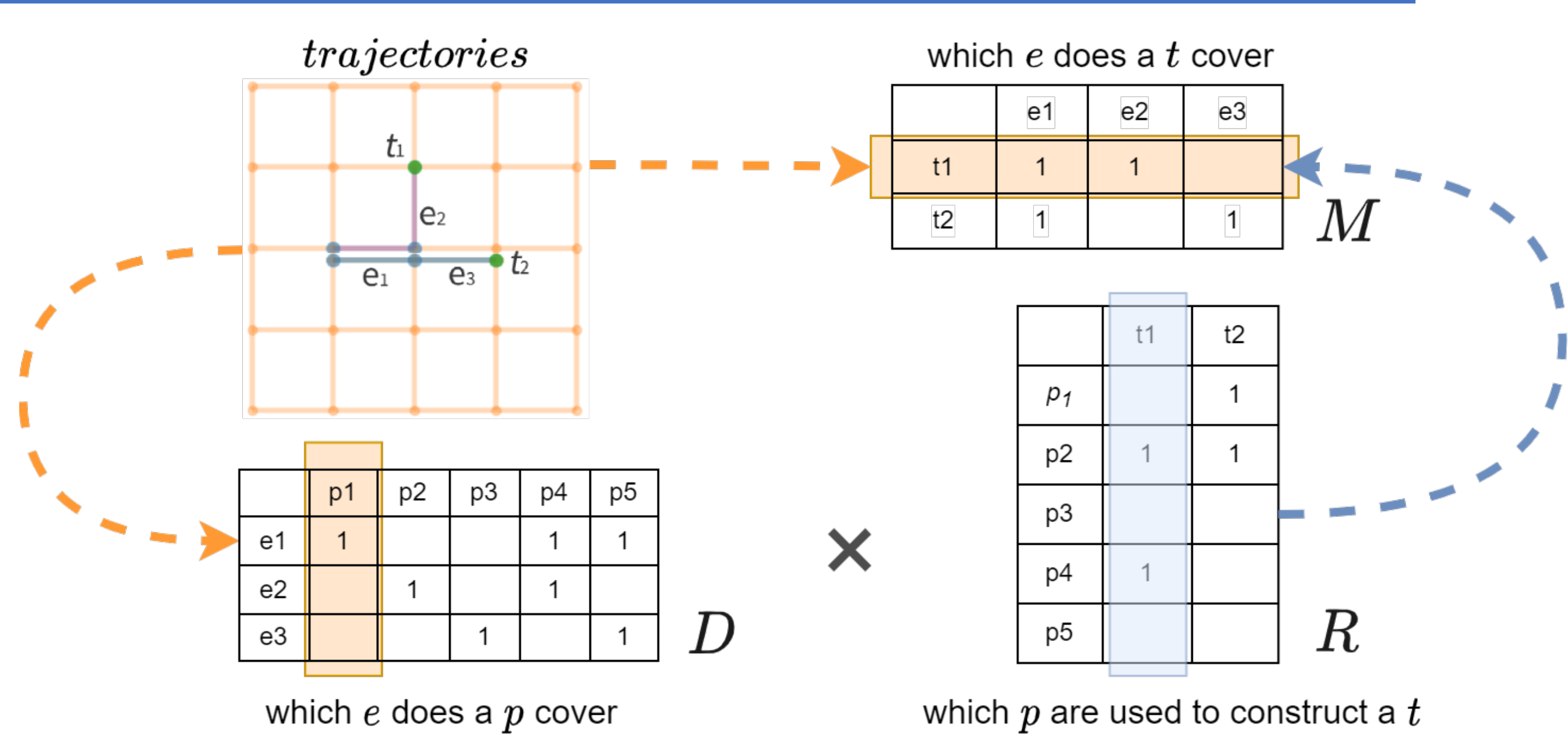


Fig 2. Represent a trajectory by concatenating pathlets from dictionary

### Evaluation criteria of a pathlet dictionary.

- This dictionary should be able to reconstruct all trajectories.
- Smaller dictionary is better, which means less redundant information.
- Average number of pathlets used to reconstruct trajectory should be as small as possible.

## Problem Formulation



- $R$  is decision matrix, corresponding to dictionary  $P$ .
- Matrix  $D$  and  $M$  record the cover relationship between  $T, E, \bar{P}$

### Problem formulation.

The size of dictionary Trade off hyper parameter

$$\min_{R_{i,j} \in \{0,1\}} \sum_i \max(R_{i,:}) + \lambda * \sum_i \sum_j |R_{i,j}|$$

$$s. t. DR \geq M$$

Constraint: each trajectory should be covered The number of pathlets used to reconstruct trajectories

## Algorithm

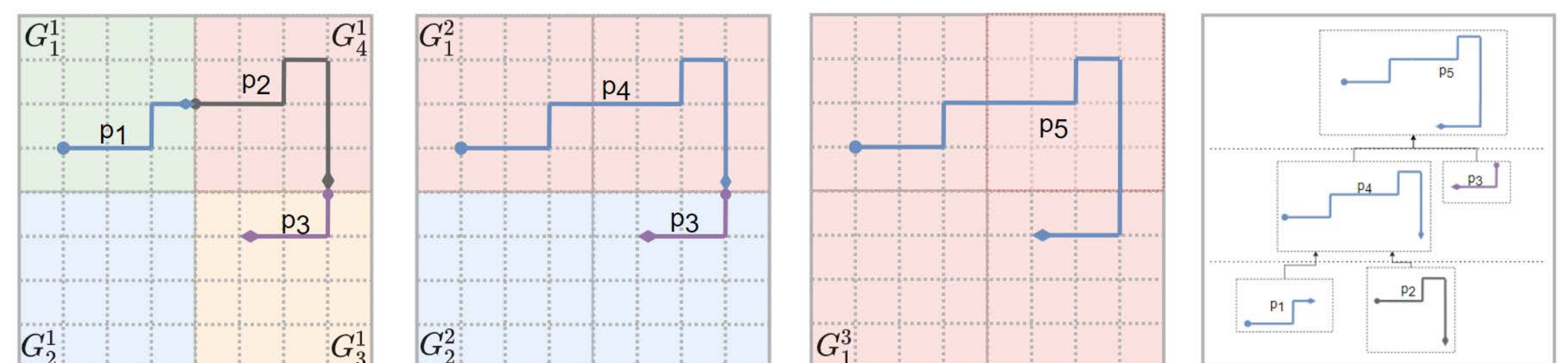
### Pathlet sparse dictionary learning with randomized rounding.

1. Relax the binary constraint and find the fractional solution  $R^*$
2. Obtain the rounded solution  $R^r$

**Probability bound.** Given the size of dataset  $|T|$ , trajectory matrix  $M$ , pathlet matrix  $D$  and trade off parameter  $\lambda$ . Then for constant parameter  $\theta$ , we have the following bound on the cost of  $R^r$ :

$$P[C(R^r) \leq 2\theta \frac{\lambda+1}{\lambda} C(R^*) \text{ and } DR^r \geq M] \geq \frac{1}{2} - |T|e^{-\theta}$$

### Hierarchical Pathlet Learning.



1. Divide the map into small grids and learn local pathlets.
2. Merge small grids and extract high-level pathlets iteratively.

## Experiment & Result

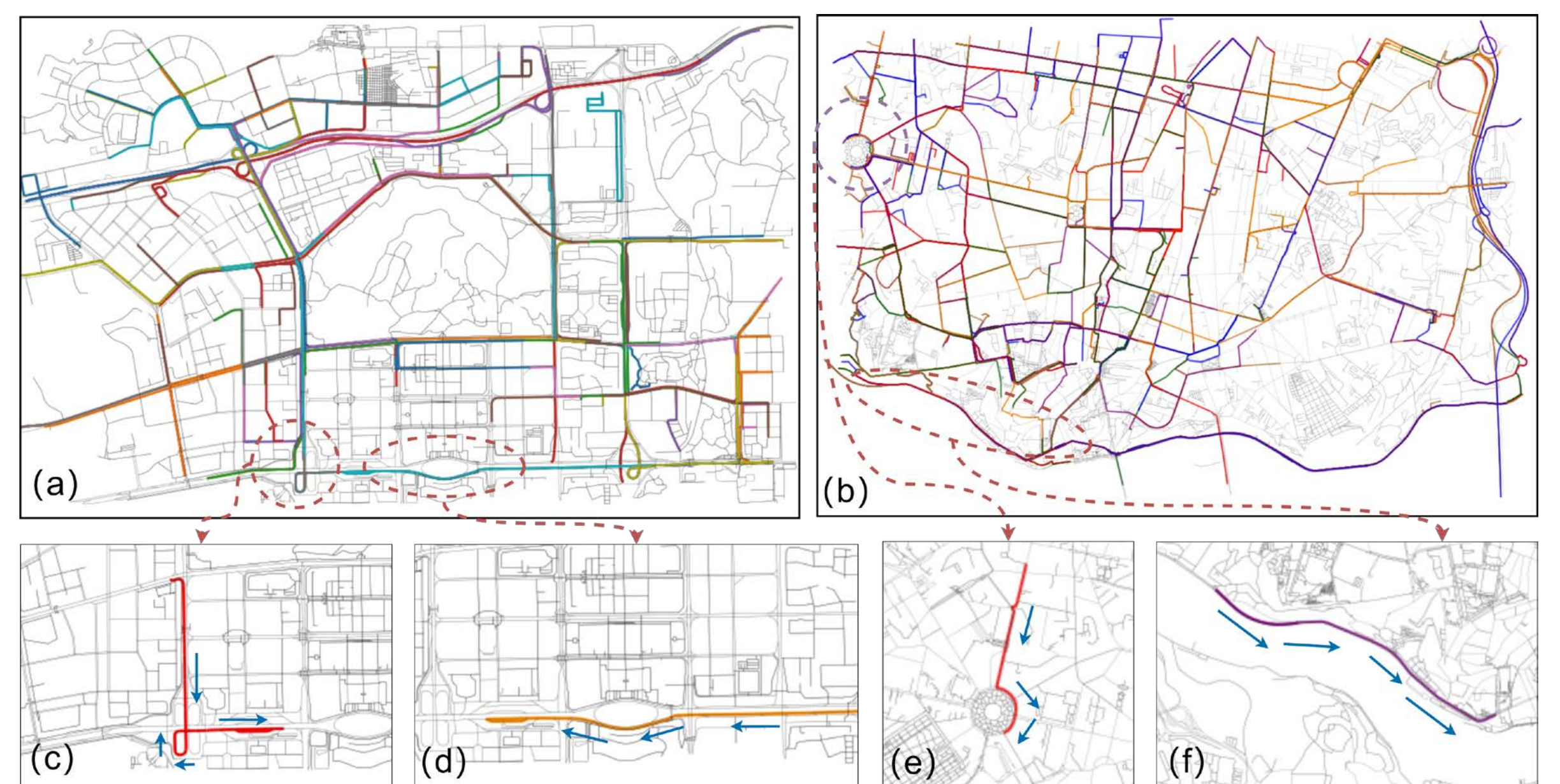


Fig 3. Visualization of Top 300 frequent pathlets.

Tab 1: The performance comparison with previous work.

Dataset	Method	Train Phrase			Test Phrase	
		dictionary size/ T	rc	cover ratio	rc	cover ratio
Porto	DP	1.79	2.14	100%	4.19	88.03%
	ours	1.02(-43.01%)	2.00(-6.54%)	99.6%	2.75	93.6%(+5.57%)
Shenzhen	DP	1.21	2.88	100%	3.01	93.9%
	ours	0.91(-36.36%)	2.75 (-4.51%)	99.1%	3.02	95.4%(+1.5%)

Tab 2. Application in Trip Time Prediction.

dataset	method	MAE	MAPE	RMSE	RMSLE
Porto	[28]	-	-	-	0.41
	[3]	171.97	-	-	-
	Ours	163.9	26.2	199.74	0.35
Porto (short trips)	[27]	39.25	14.74	52.35	-
	Ours	32.87	9.37	37.59	0.08

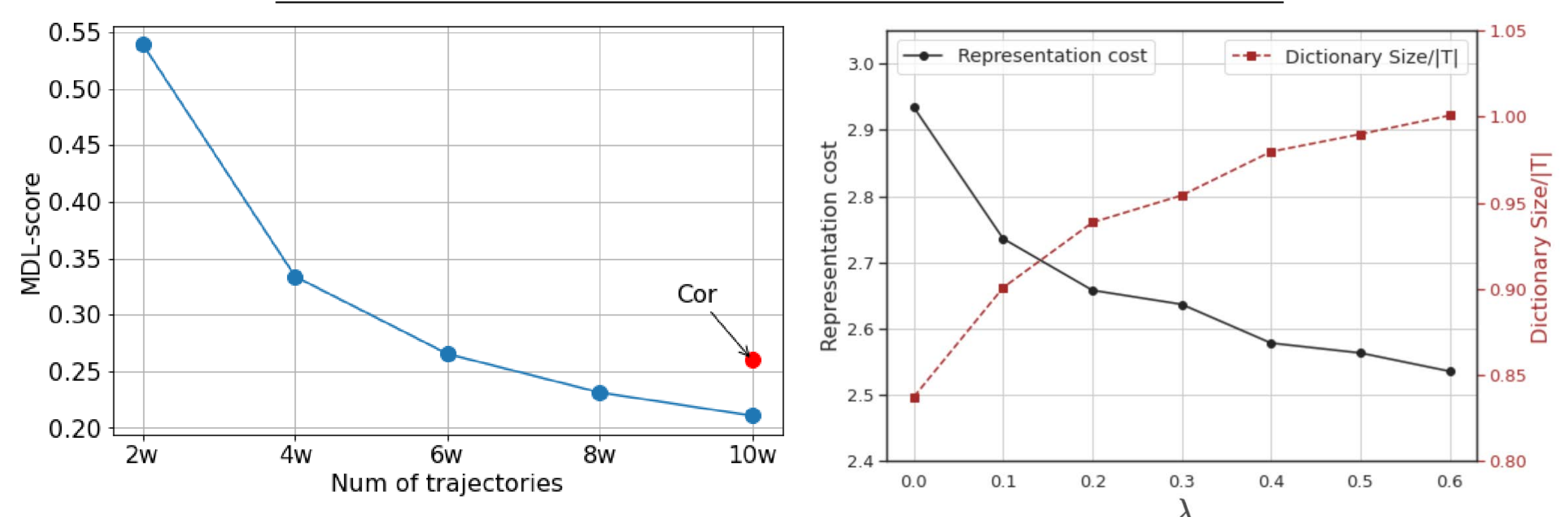


Fig 4. Application in Data Compression. Fig 5. Effect of hyper-parameter.

## Conclusion

- A dictionary learning based method with theoretical probability bound analysis is proposed to solve the trajectory representation problem.
- The most frequent pathlets generated are visualized.
- The numerical experiment result demonstrates the effectiveness and broad prospects in downstream tasks.