$w = \begin{bmatrix} w \\ b \\ \xi \end{bmatrix}$

KKT conditions

Stationary: $\frac{\partial L}{\partial w} = 0 \Rightarrow w^* = \sum_{i=1}^{m} \alpha_i^* y^i x^i$

$\frac{\partial L}{\partial b} = 0 \Rightarrow \sum_{i=1}^{m} \alpha_i y^i = 0$

$\frac{\partial L}{\partial \xi} = 0 \Rightarrow \alpha_i = C - r_i$ (1)

Complementary: $\alpha_i g_i(\vec{w}) = 0 \Rightarrow \alpha_i (y^i(w^T x^i + b) - 1 + \xi_i) = 0$ (2)

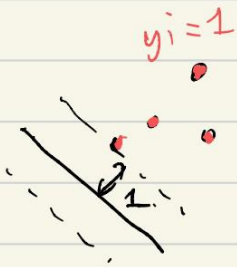
$r_i \bar{g}_i(\vec{w}) = 0 \Rightarrow r_i \xi_i = 0$ (3)

dual feasibility: $\alpha_i \geq 0$ (4)

$r_i \geq 0$ (5)

primal feasibility: $g_i(\vec{w}) \leq 0 \Rightarrow y^i(w^T x^i + b) - 1 + \xi_i \geq 0$ (6)

$\bar{g}_i(\vec{w}) \leq 0 \Rightarrow \xi_i \geq 0$ (7)



Case 1 $\alpha_i = 0$.

By (1), $\alpha_i = C - r_i$, then $r_i = C$

Since $C > 0 \Rightarrow r_i > 0$.

By (3) $r_i \xi_i = 0 \Rightarrow \xi_i = 0$

$y^i(w^T x^i + b) - 1 \geq 0$.

$y^i(w^T x^i + b) \geq 1$

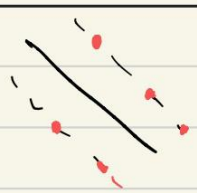
$x^{(i)}$ is on the correct side of the margin!

Case 2 $\alpha_i \neq 0$, & $\alpha_i < C$

$r_i = C - \alpha_i$, $\alpha_i < C$, $r_i > 0$.

By (3), $r_i \xi_i = 0 \Rightarrow \xi_i = 0$, since $\alpha_i \neq 0$, $\alpha_i \geq 0$.

$\alpha_i > 0$: by (2), $y^i(w^T x^i + b) - 1 + \xi_i = 0$.

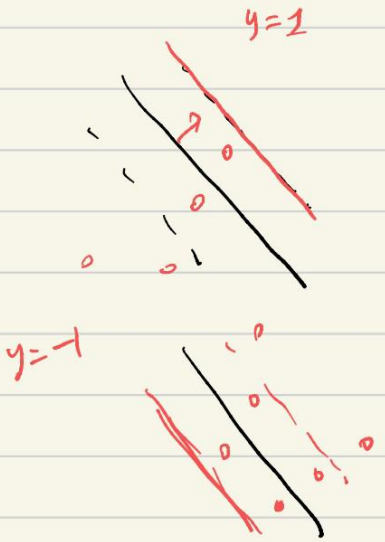


$y^i(w^T x^i + b) = 1$.

$x^{(i)}$ on the margin!

Case 3 $\alpha_i \neq 0$, $\alpha_i = C$

Case 3 $\alpha_i \neq 0,\ \alpha_i = C$



Since $\alpha_i = C$, $r_i = C - \alpha_i = 0$.

Since $r_i \xi_i \geq 0 \Rightarrow \xi_i \geq 0$.

Since $\alpha_i = C > 0$,

$$y^i (w^T x^i + b) = 1 + \xi = 0.$$

$$y^i (w^T x^i + b) \leq 1.$$

$x^{(i)}$ are on the wrong side of the margin.