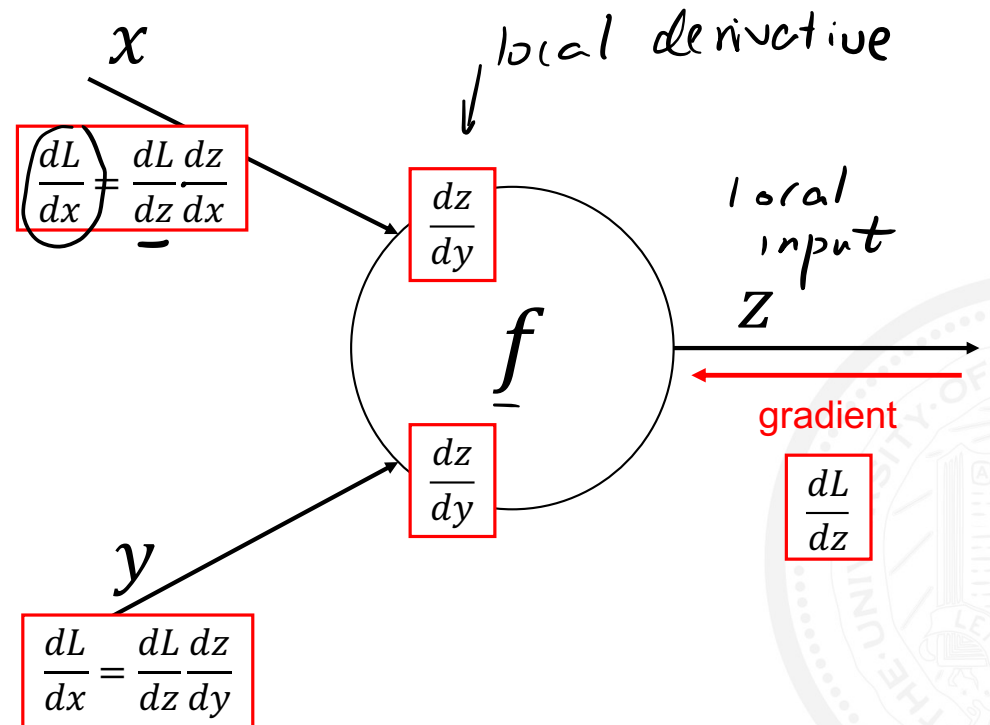
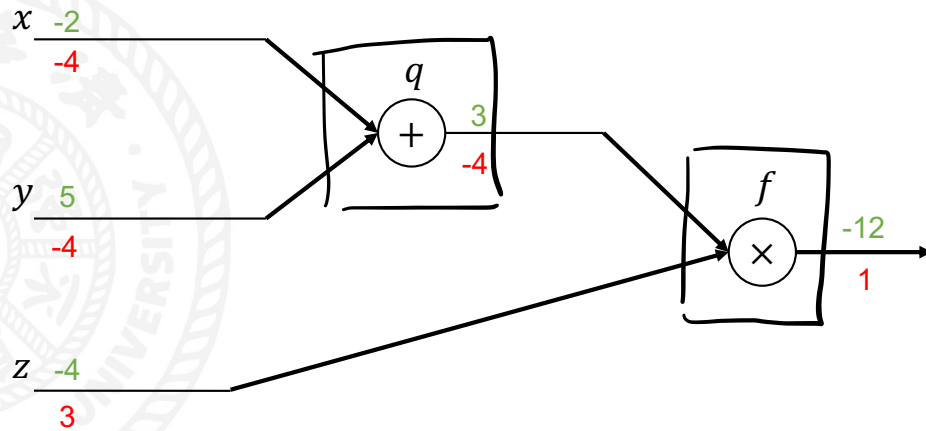


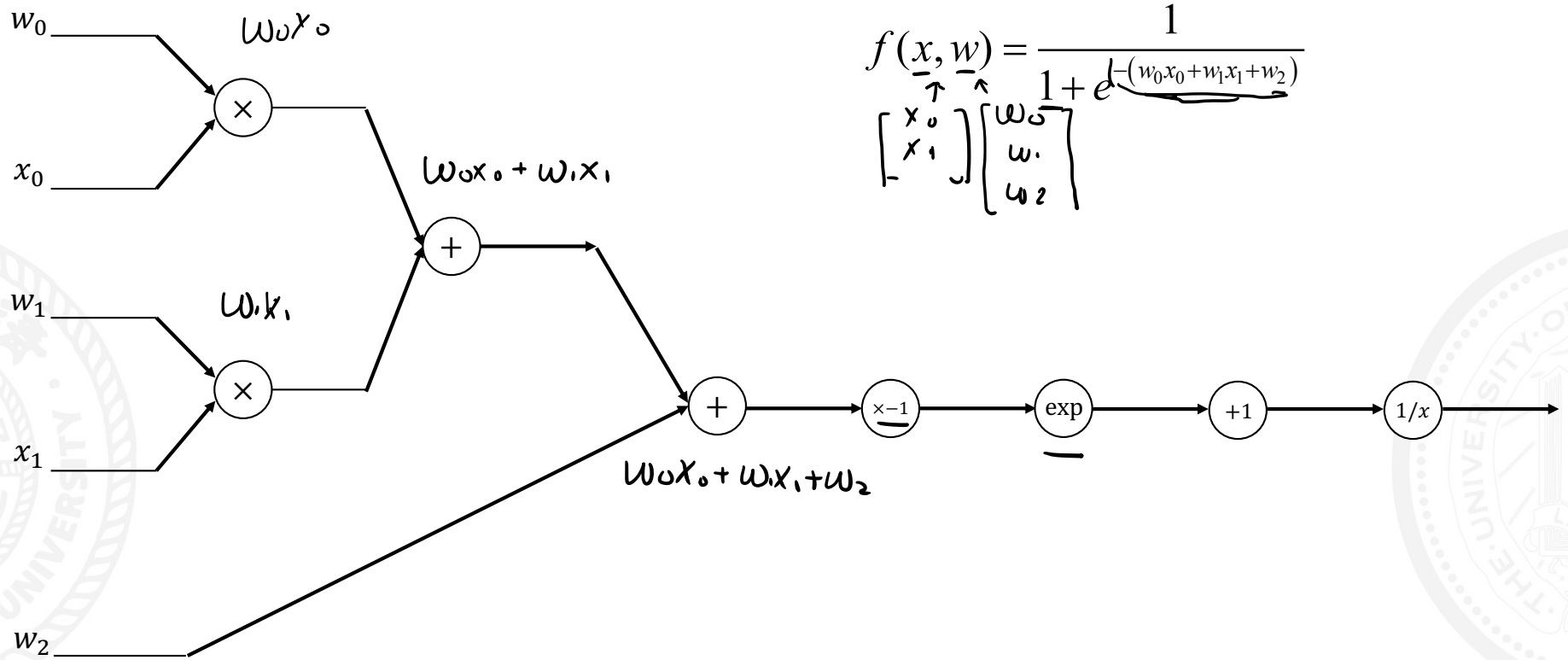
Backpropagation

$$f(x, y, z) = (x + y)z$$

We want $\frac{df}{dx}, \frac{df}{dy}, \frac{df}{dz}$

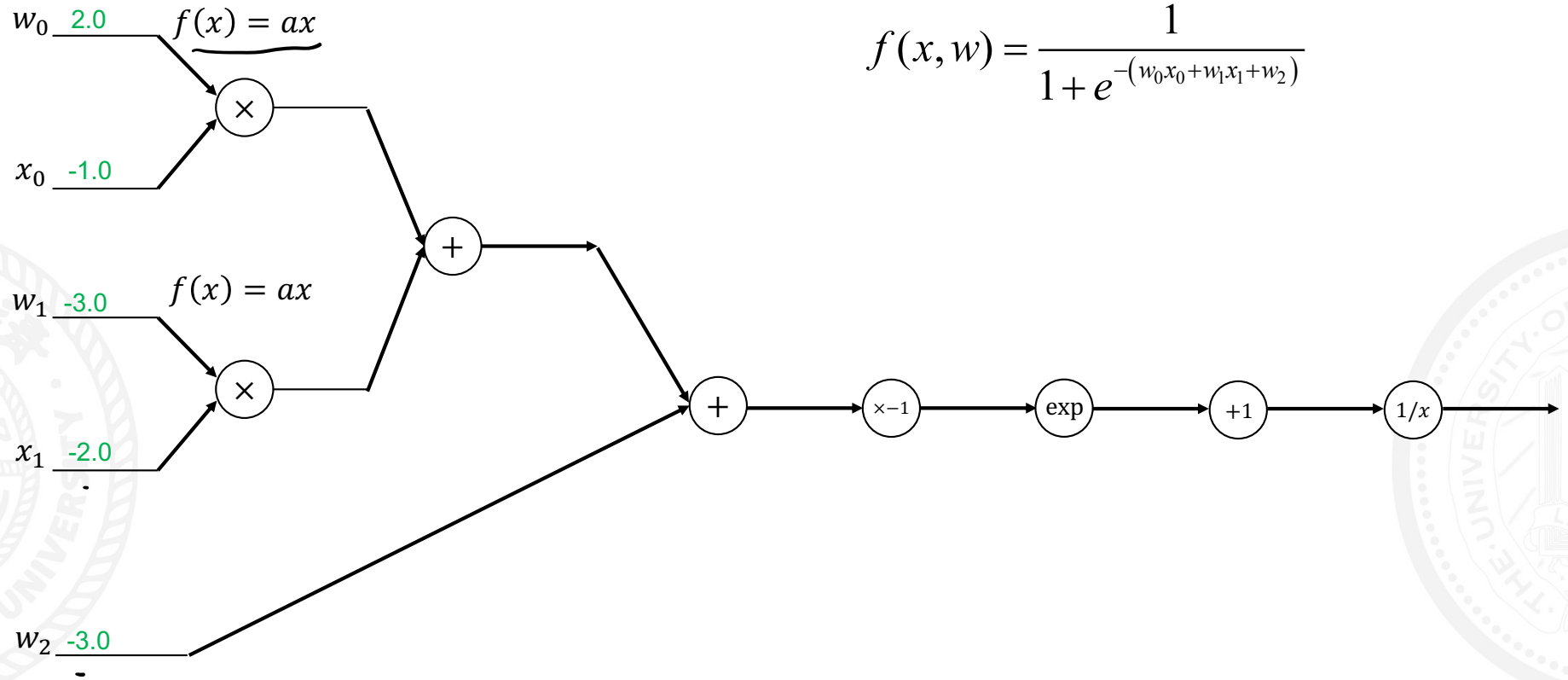


Backpropagation

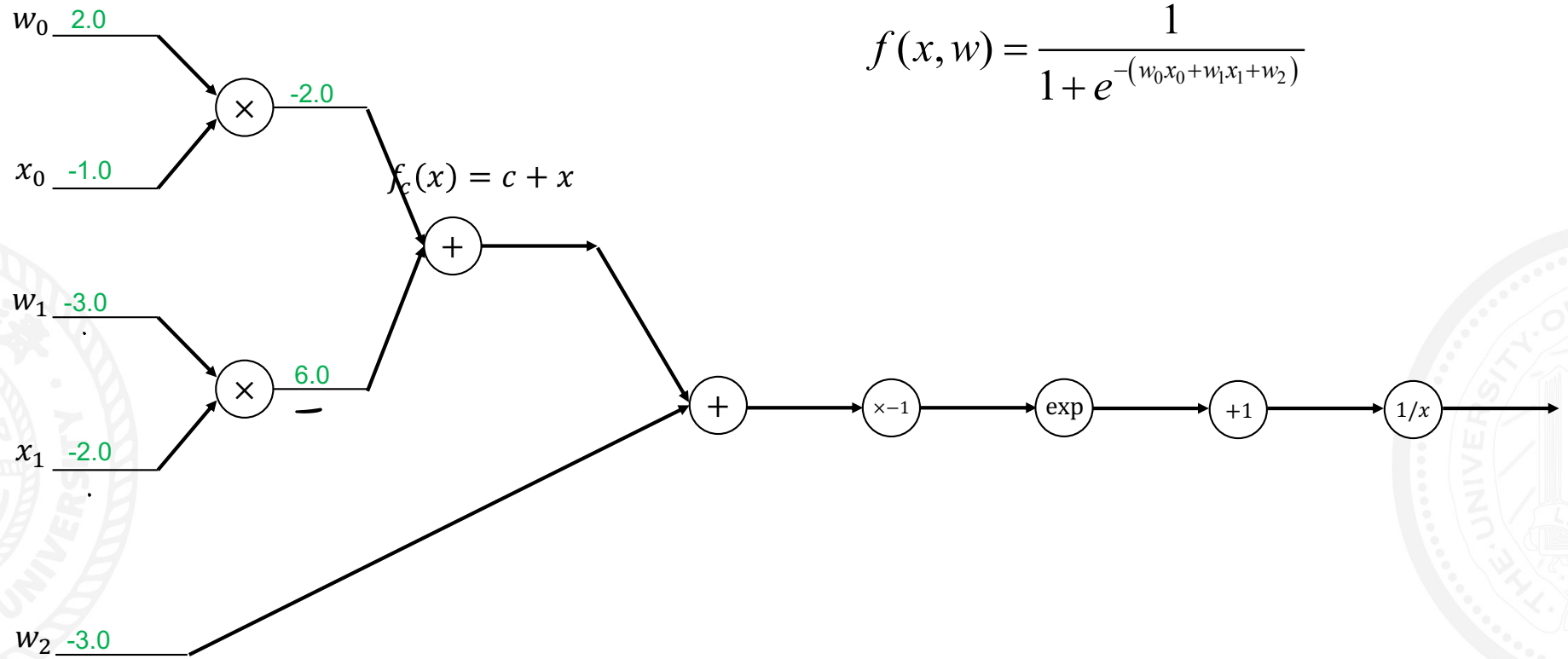


$$f(\underline{x}, \underline{w}) = \frac{1}{1 + e^{-(w_0 x_0 + w_1 x_1 + w_2)}}$$

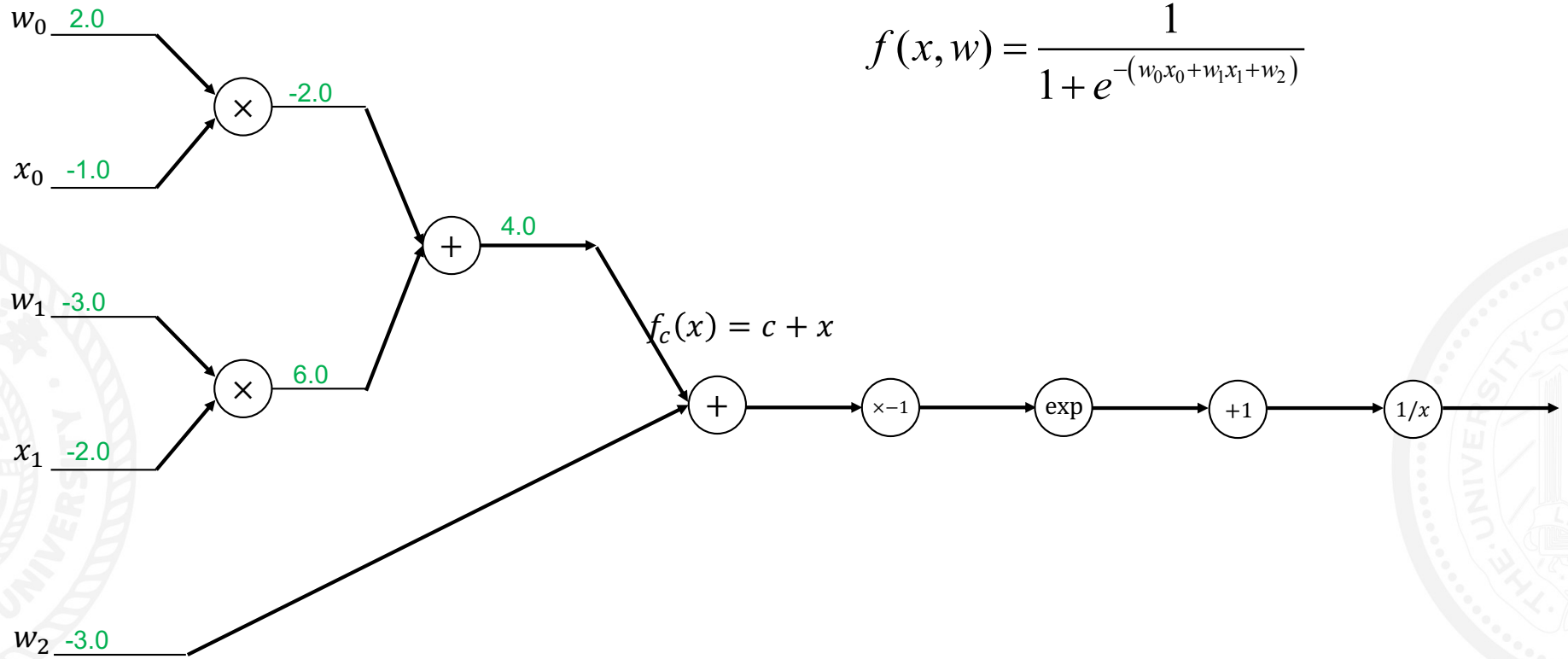
Backpropagation



Backpropagation

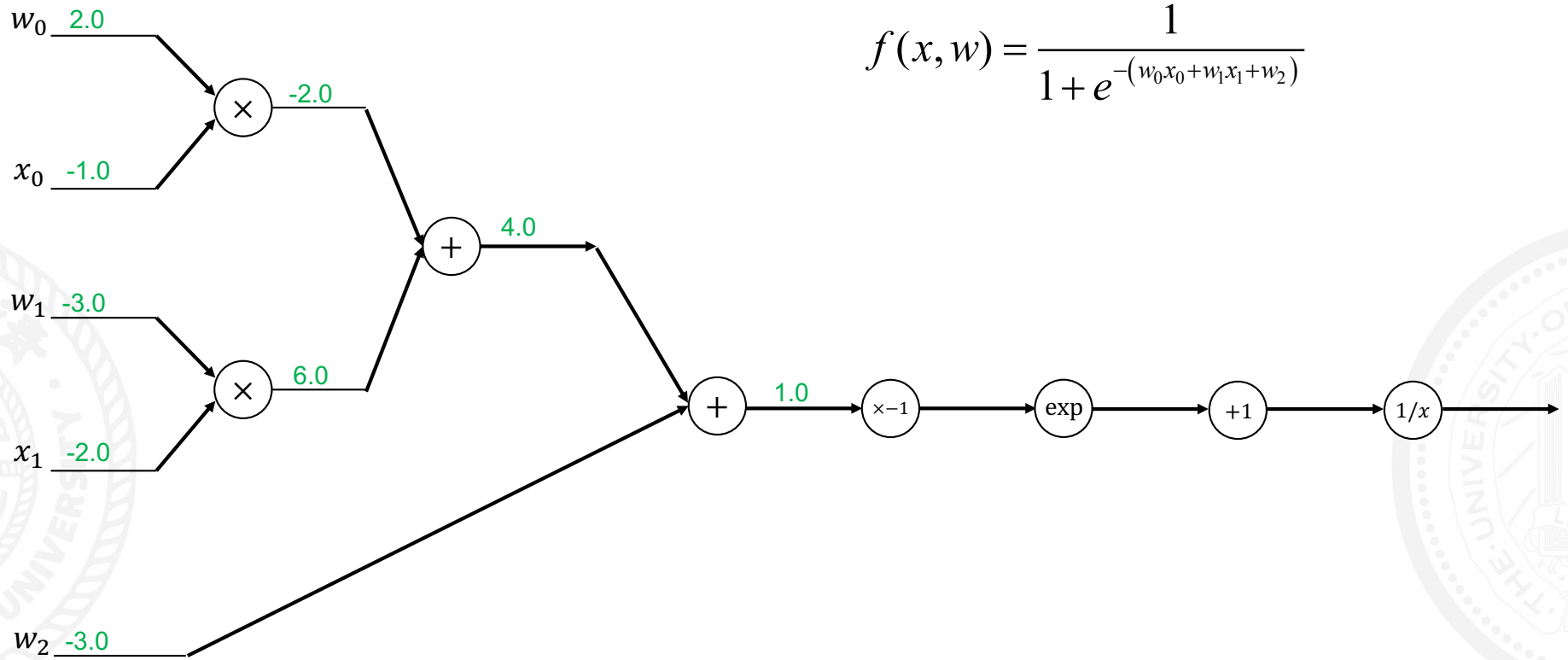


Backpropagation



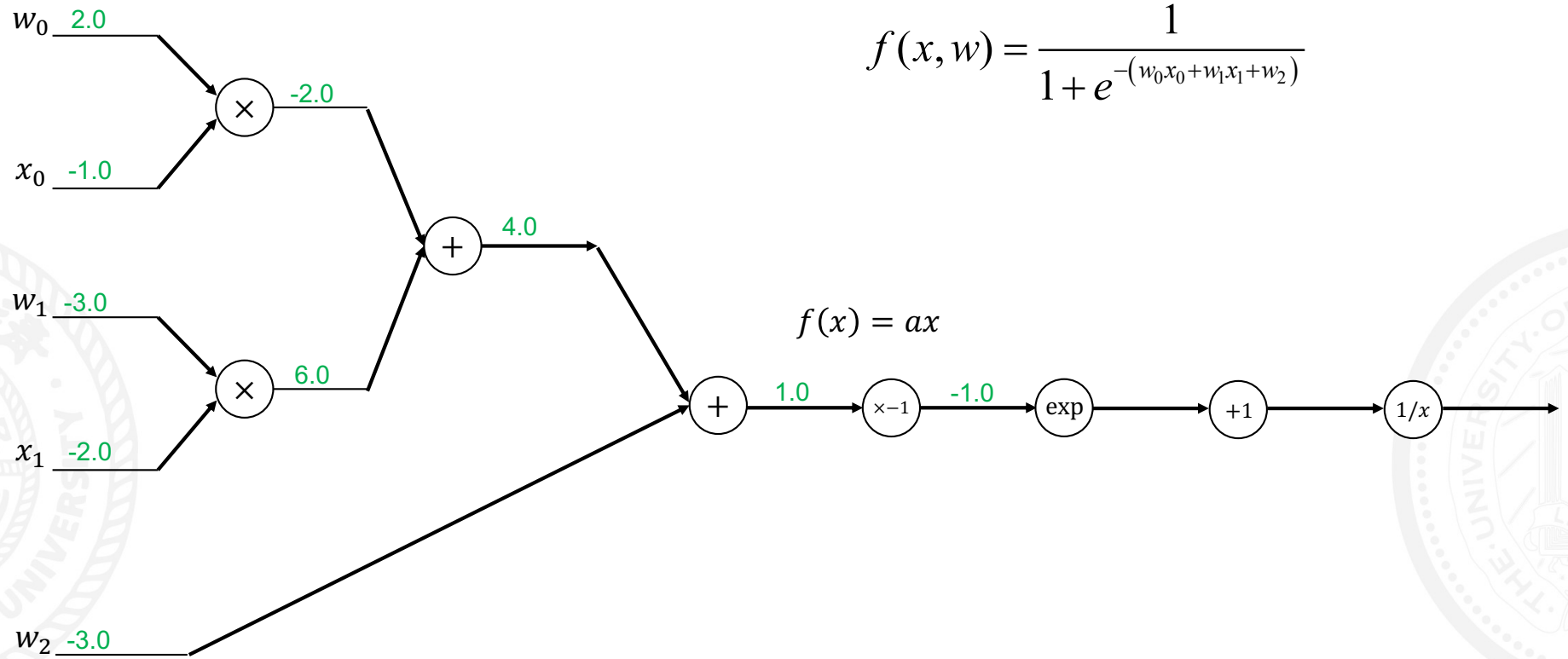
$$f(x, w) = \frac{1}{1 + e^{-(w_0 x_0 + w_1 x_1 + w_2)}}$$

Backpropagation

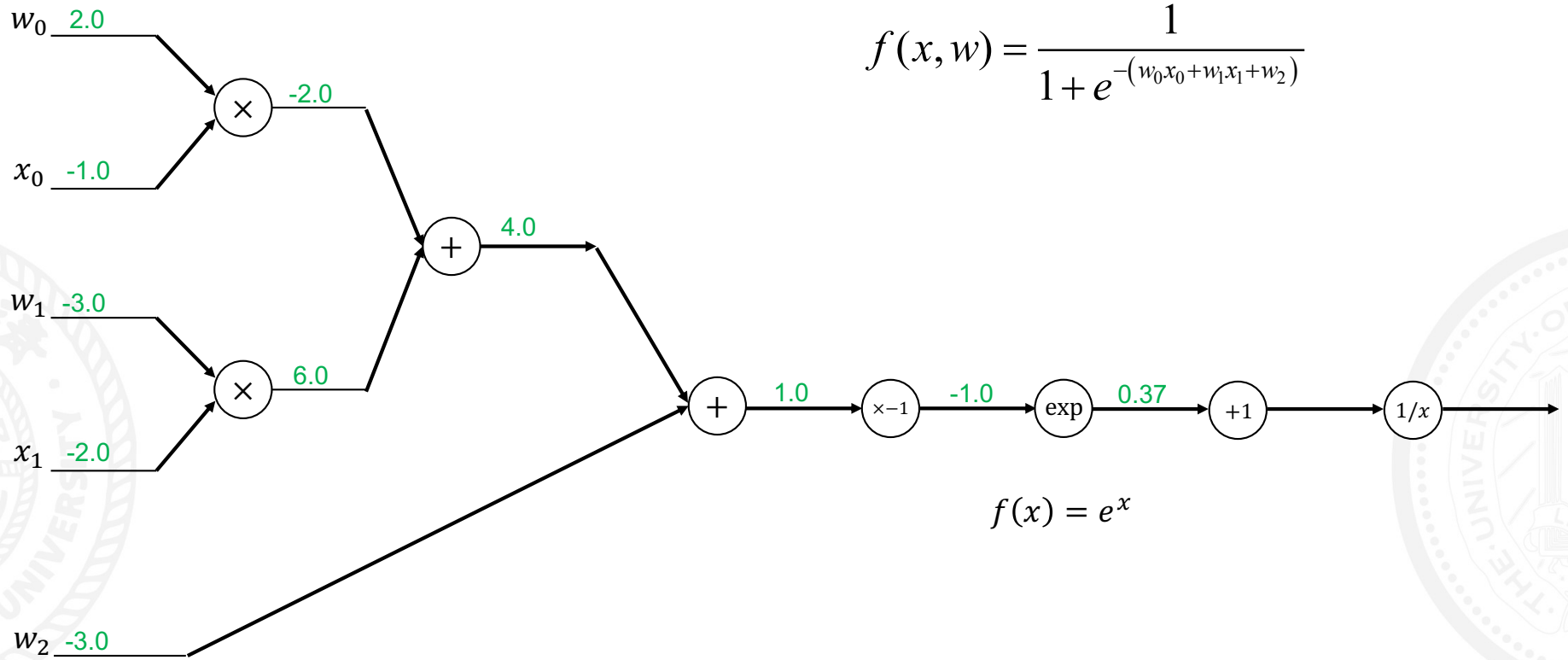


$$f(x, w) = \frac{1}{1 + e^{-(w_0 x_0 + w_1 x_1 + w_2)}}$$

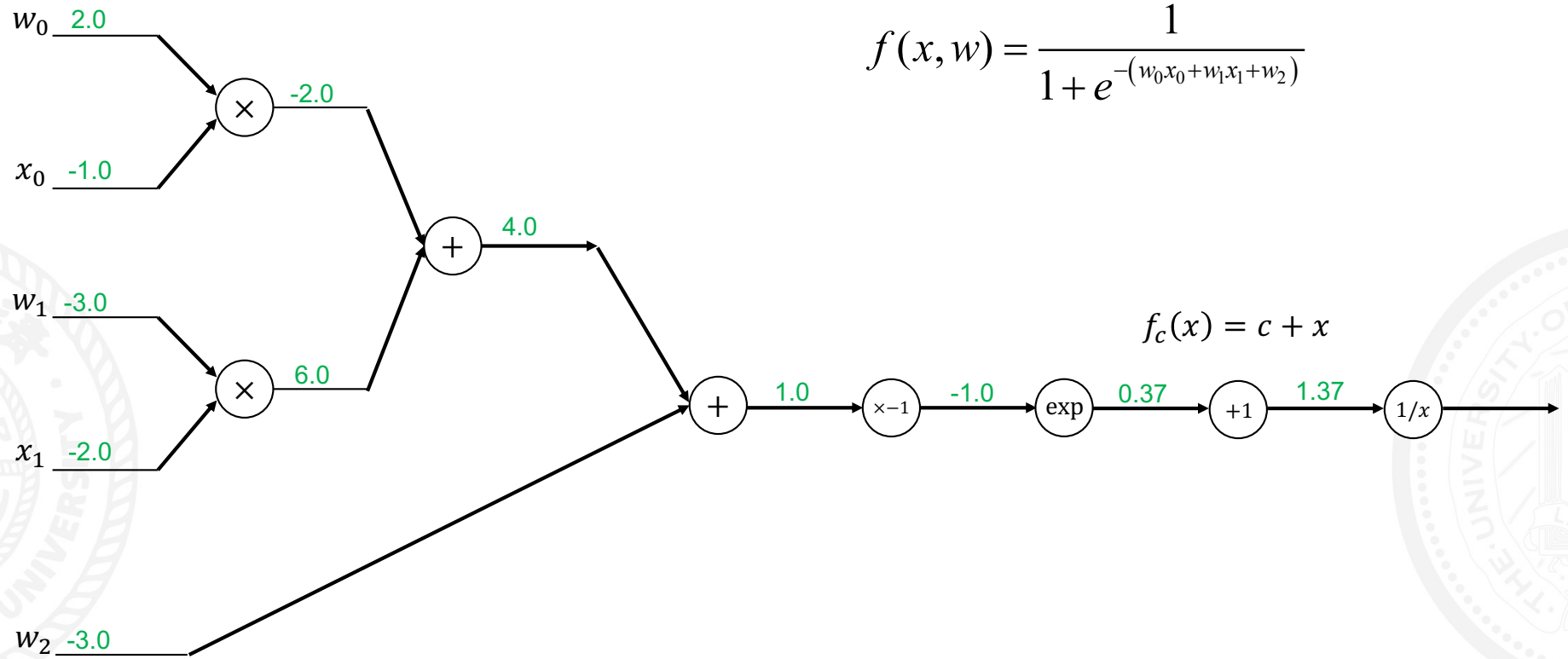
Backpropagation



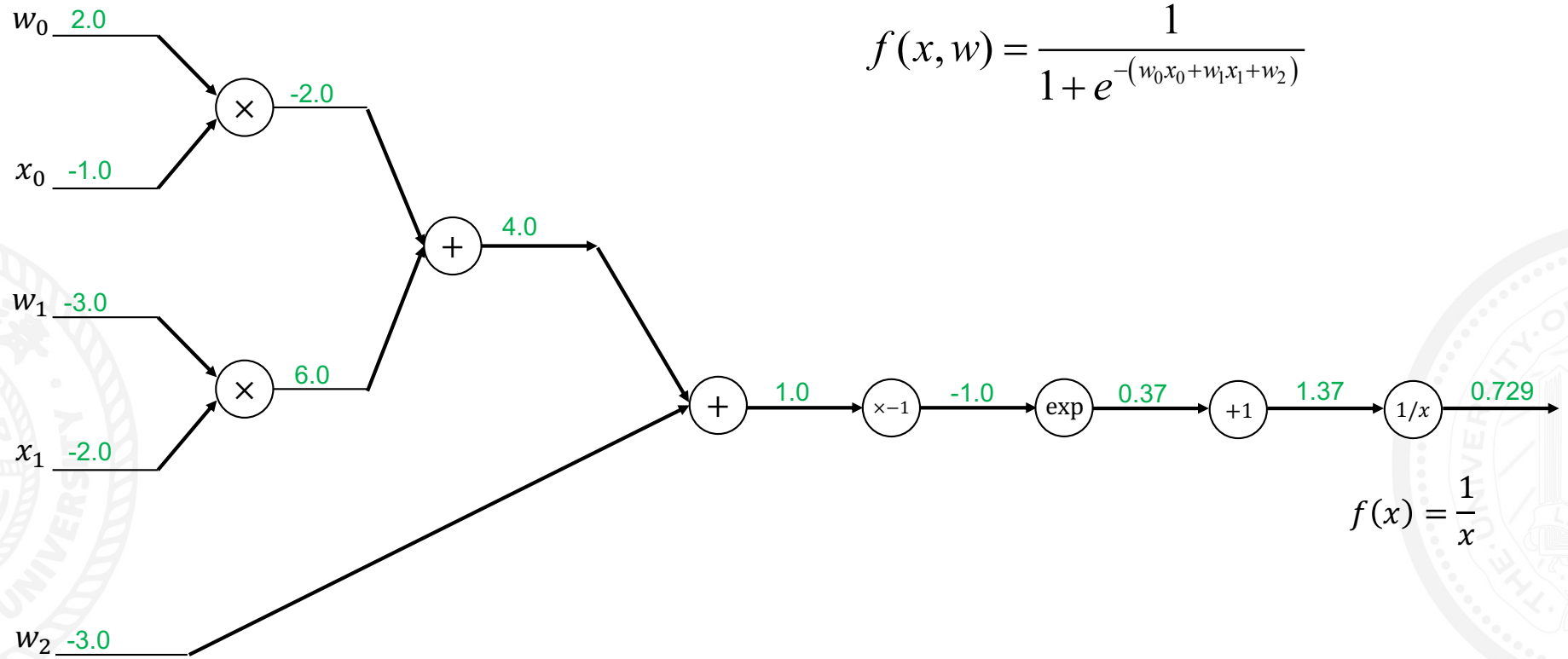
Backpropagation



Backpropagation



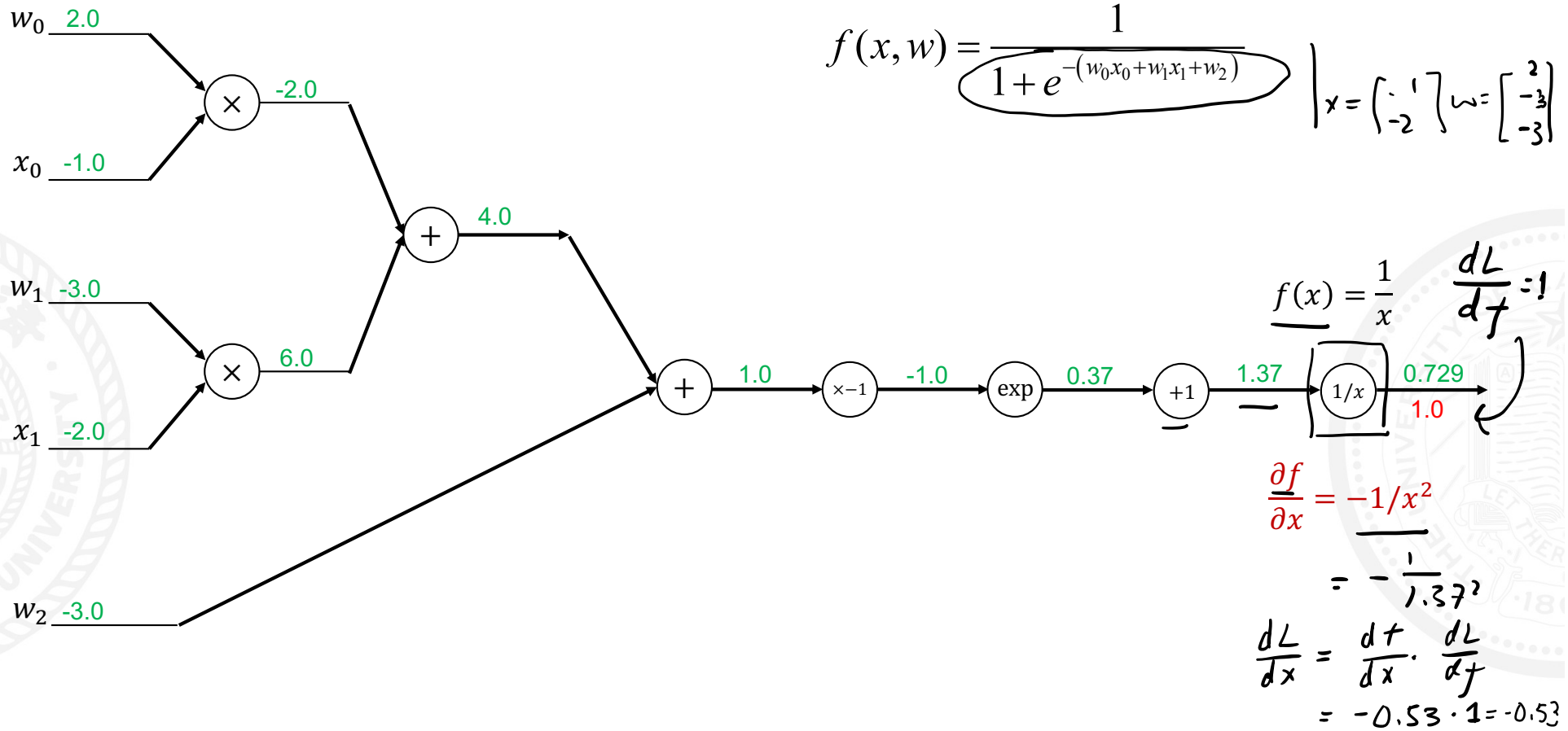
Backpropagation



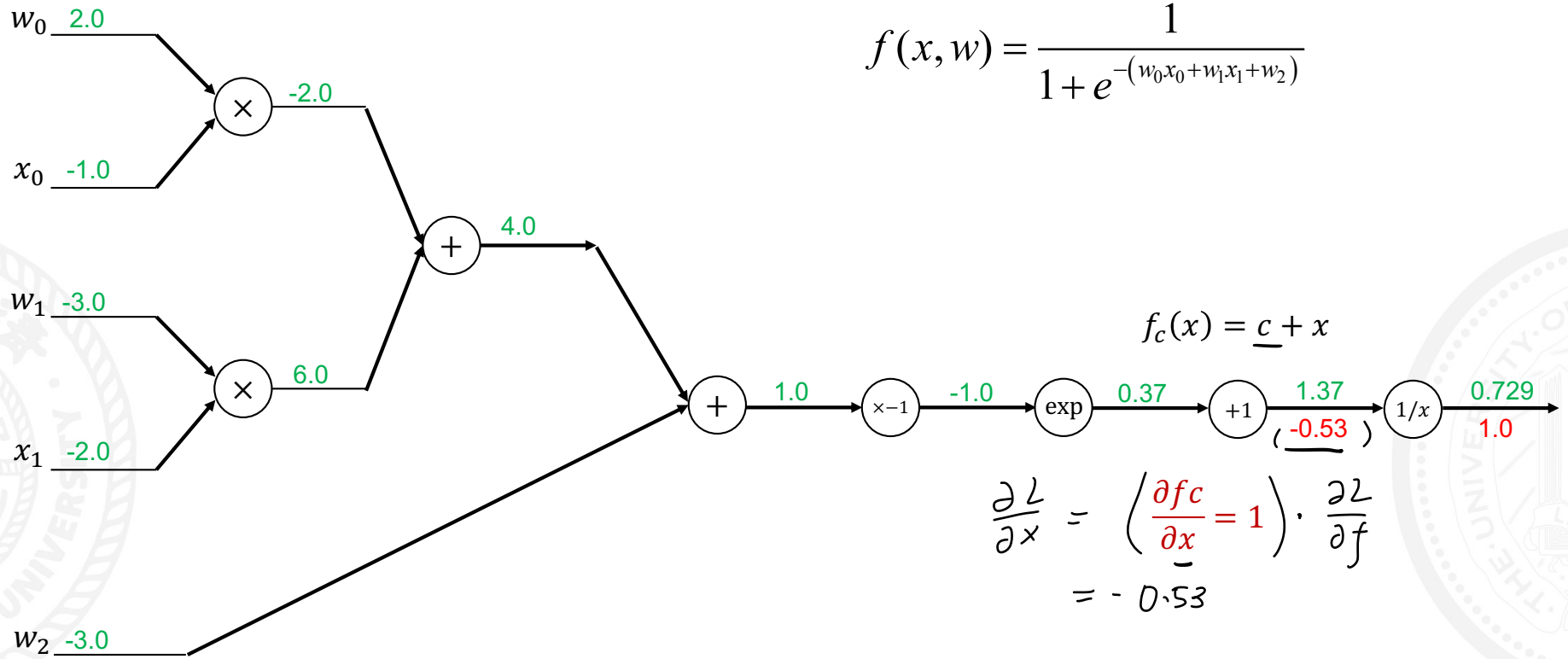
$$f(x, w) = \frac{1}{1 + e^{-(w_0 x_0 + w_1 x_1 + w_2)}}$$

$$f(x) = \frac{1}{x}$$

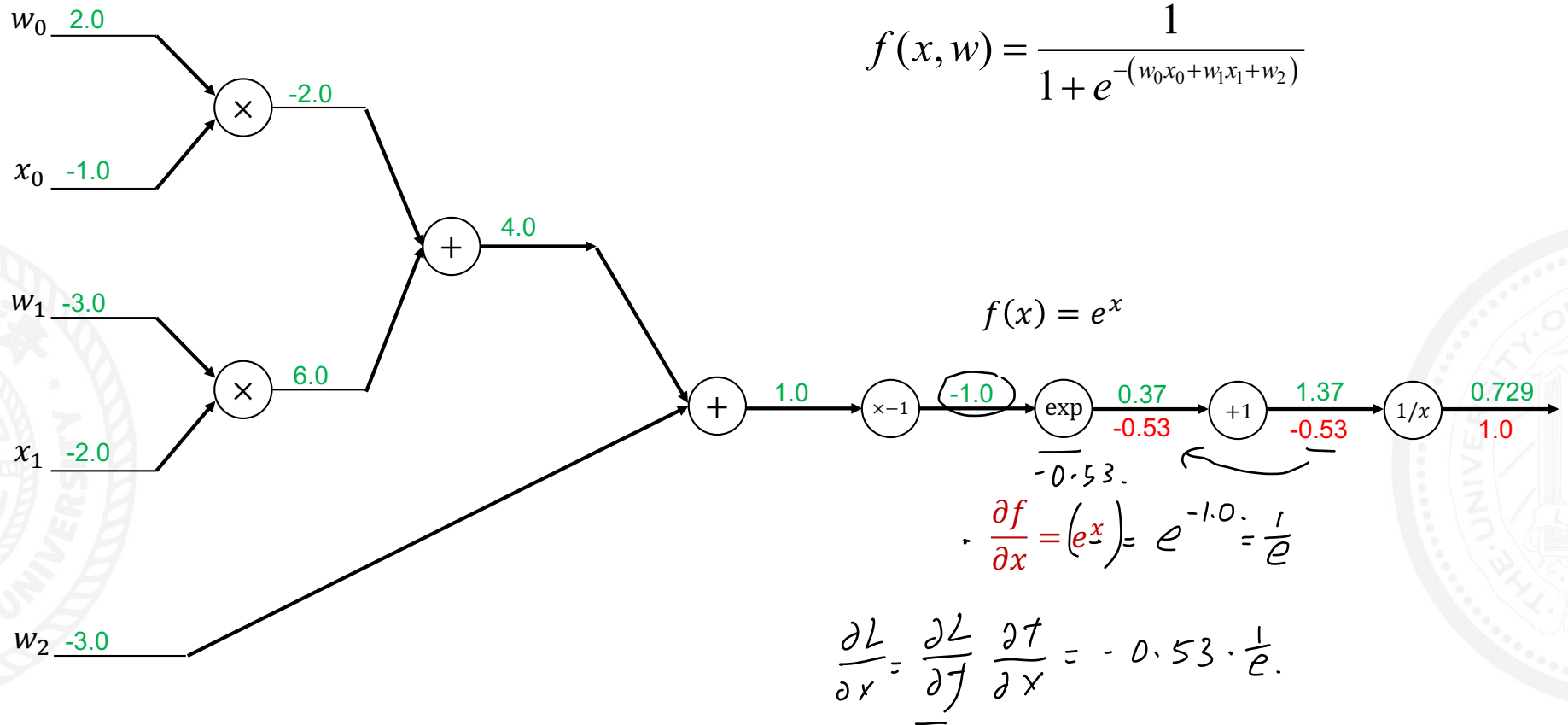
Backpropagation



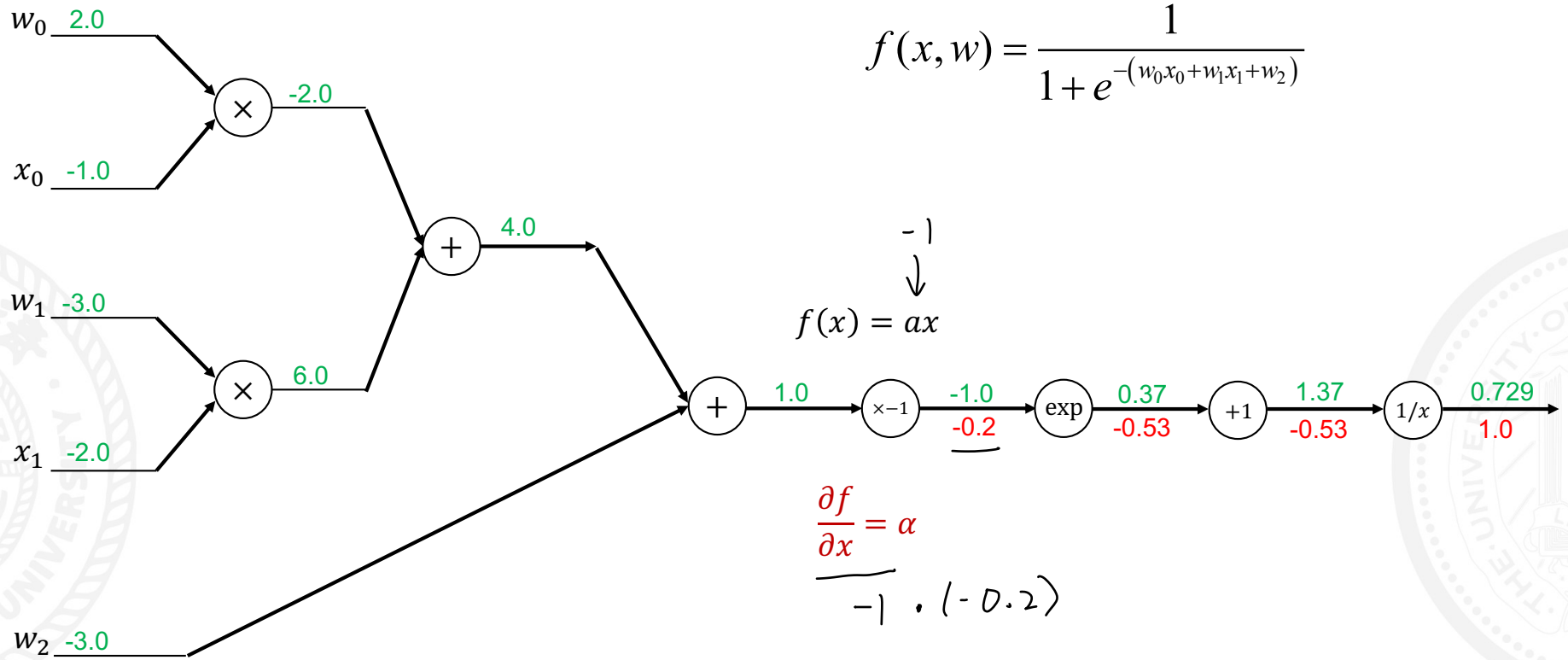
Backpropagation



Backpropagation



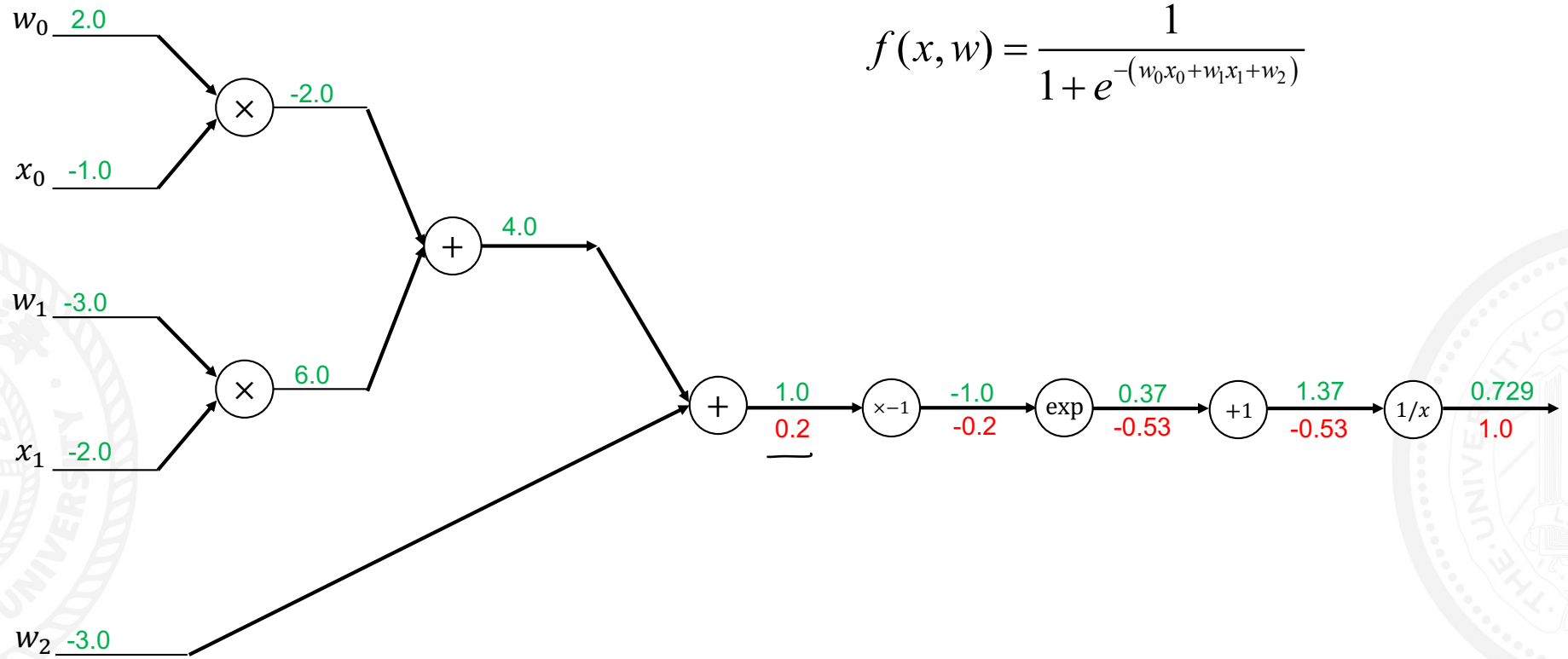
Backpropagation



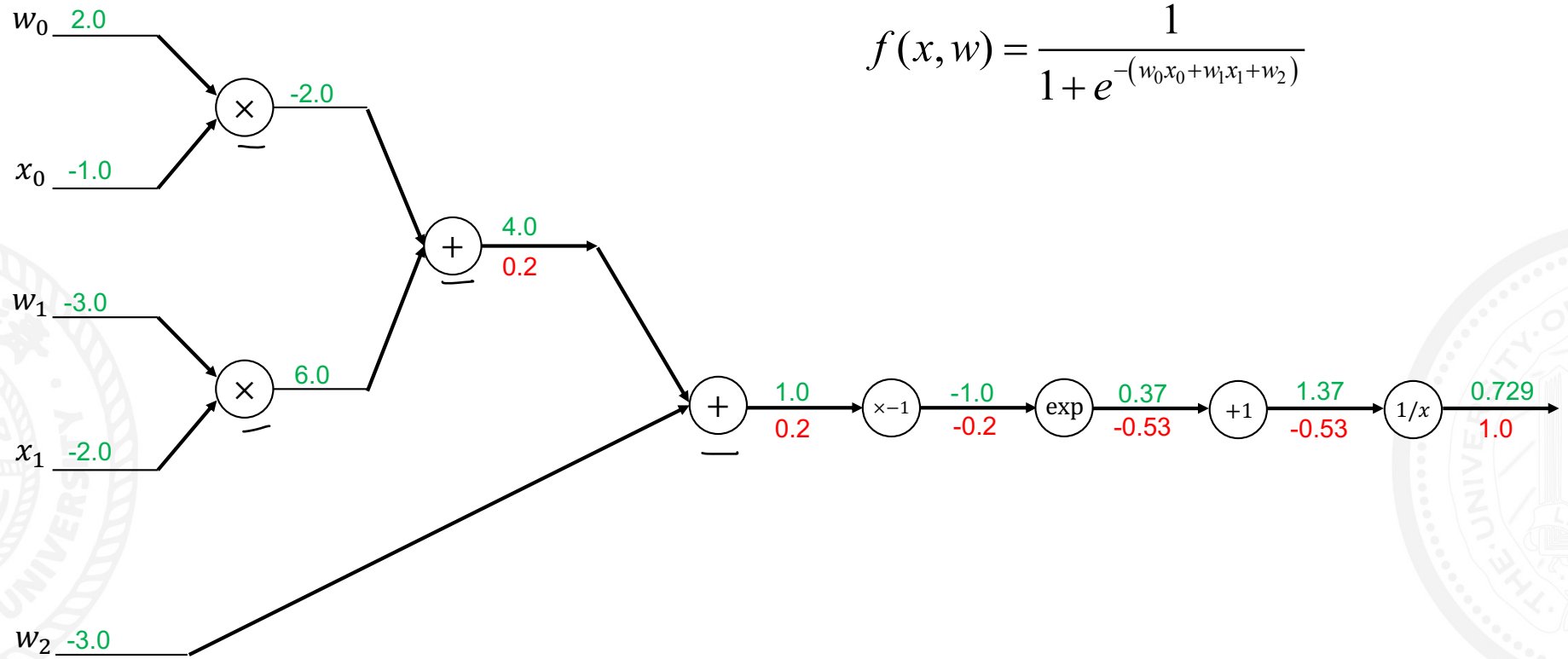
$$f(x, w) = \frac{1}{1 + e^{-(w_0 x_0 + w_1 x_1 + w_2)}}$$

$$f(x) = ax$$

Backpropagation

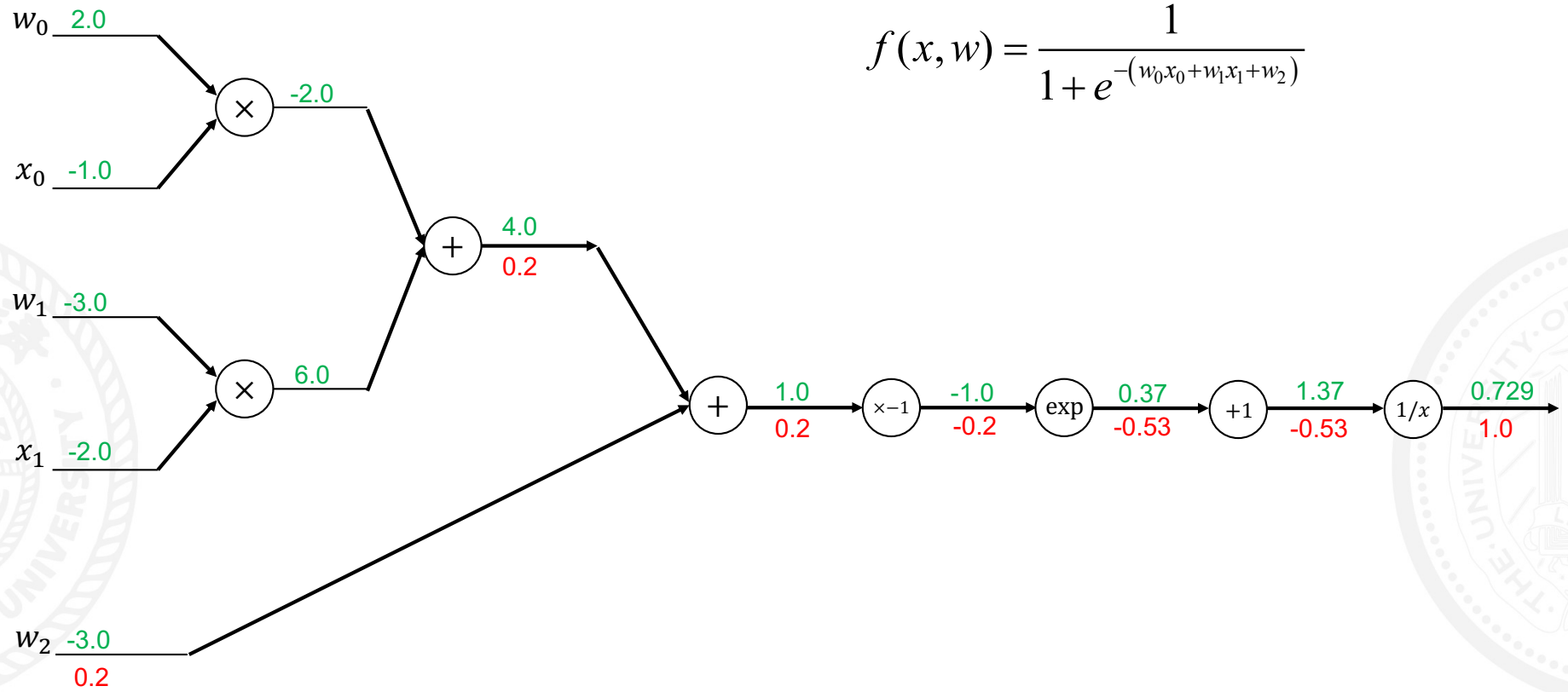


Backpropagation

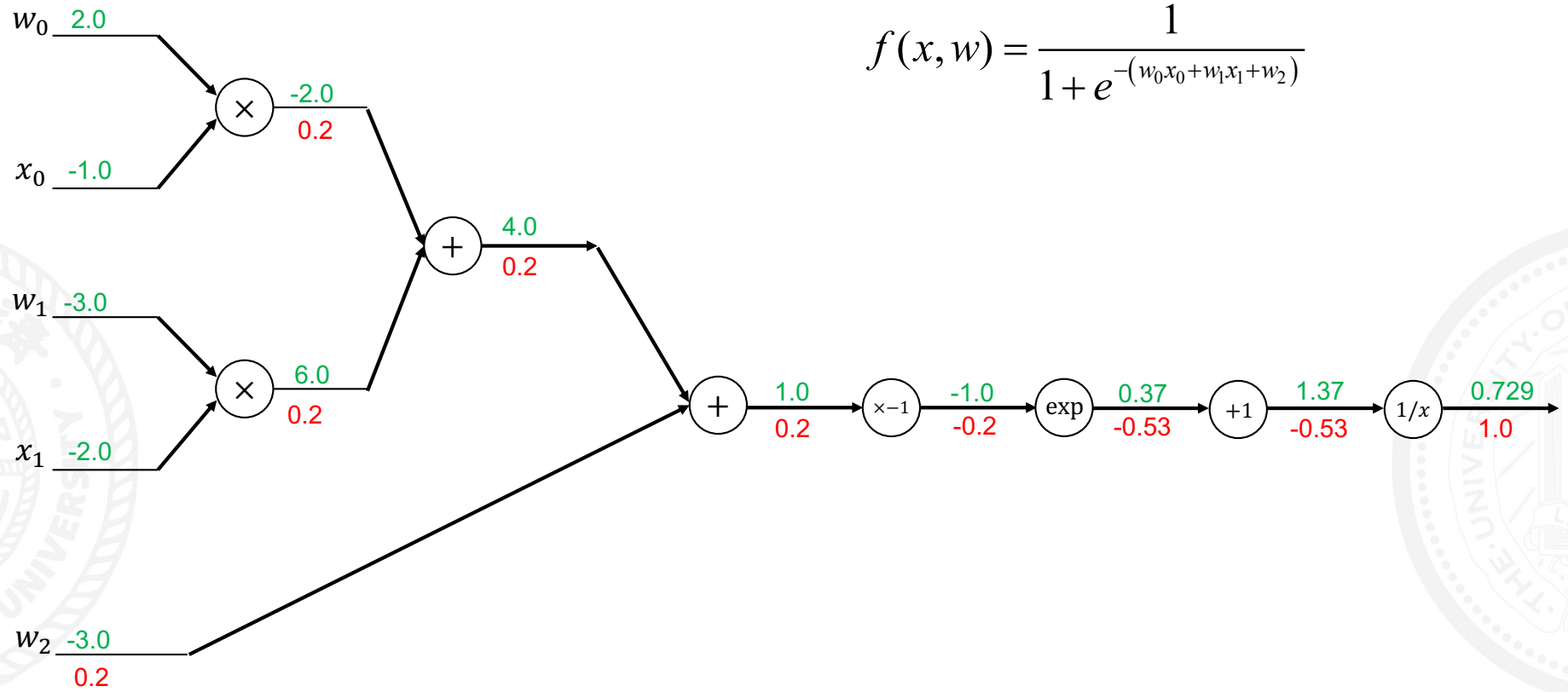


$$f(x, w) = \frac{1}{1 + e^{-(w_0 x_0 + w_1 x_1 + w_2)}}$$

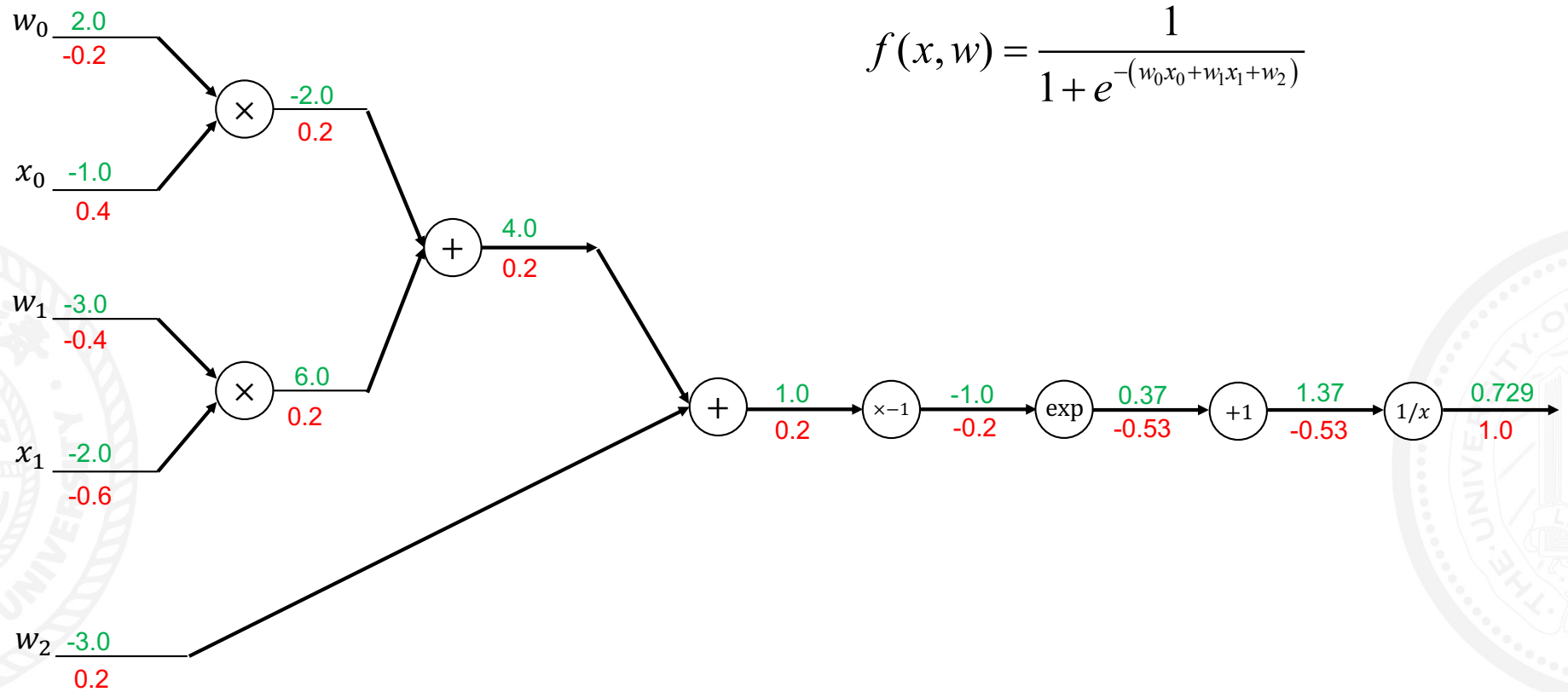
Backpropagation



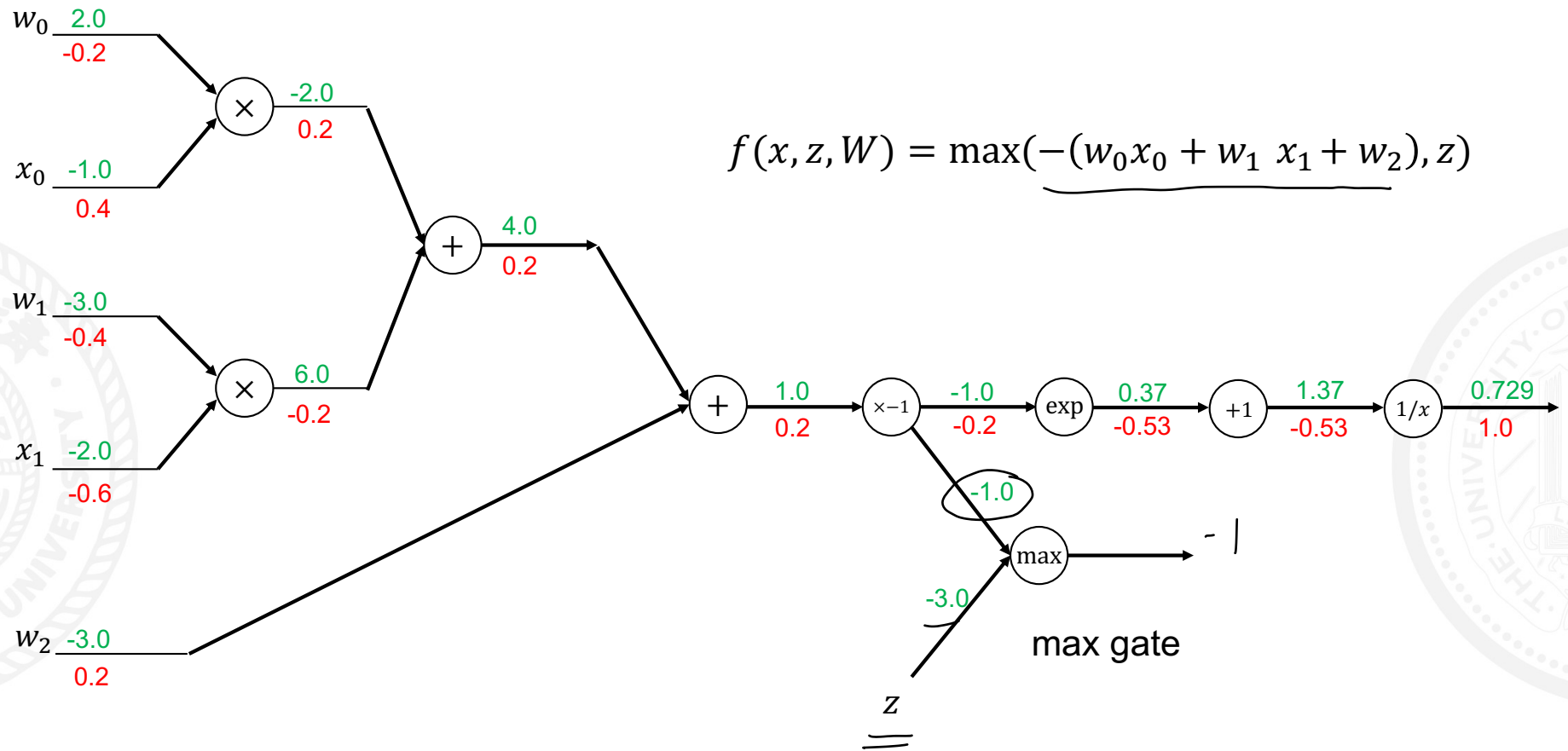
Backpropagation



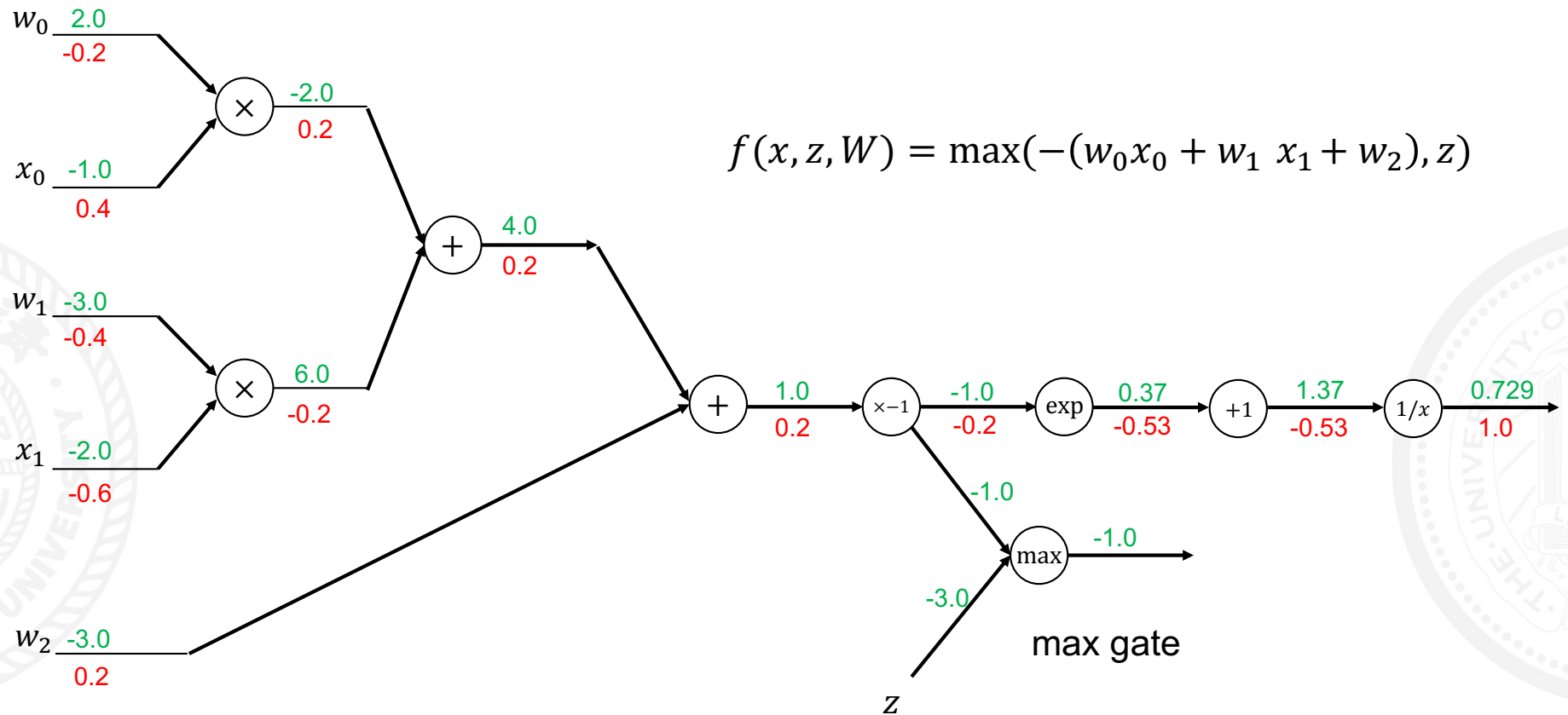
Backpropagation



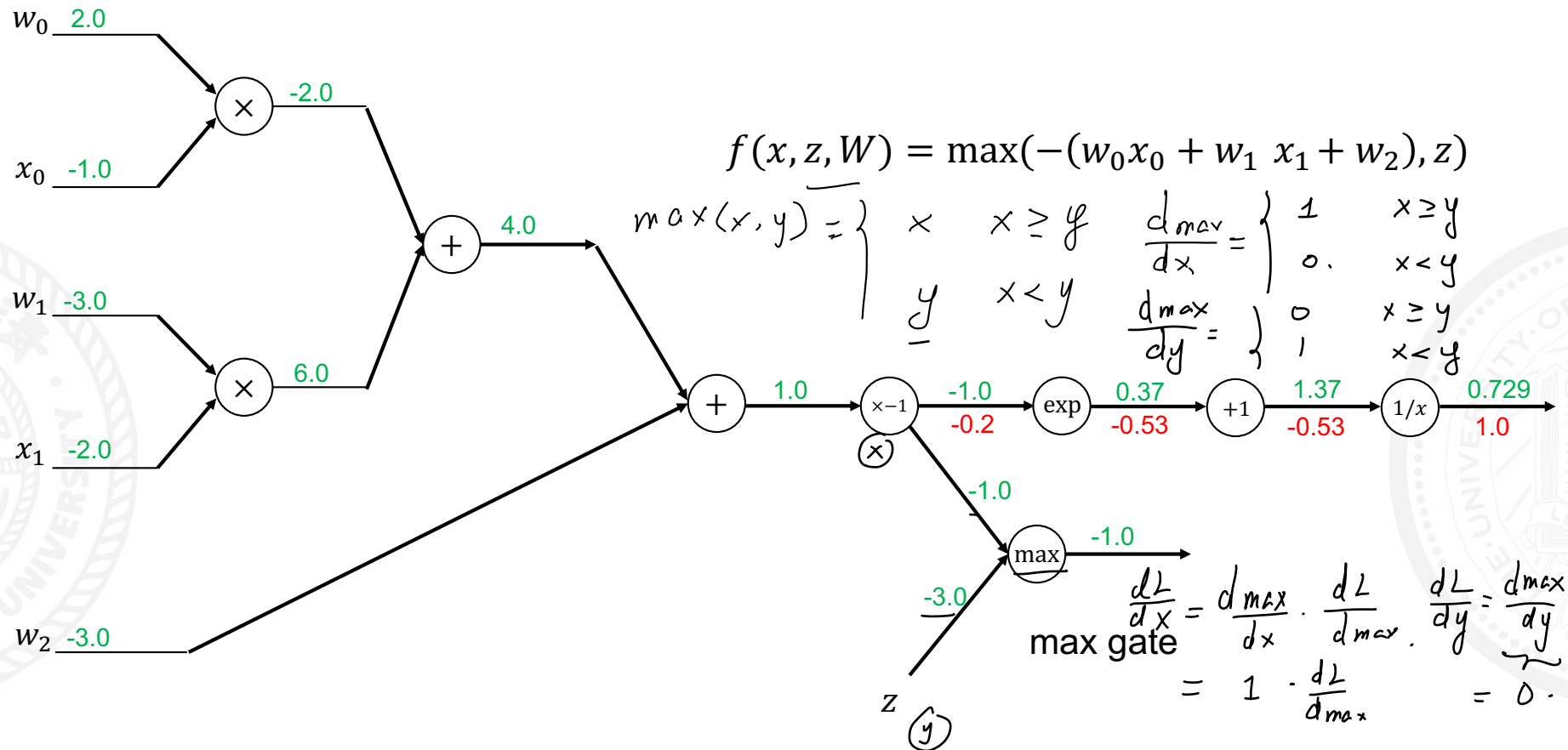
Backpropagation



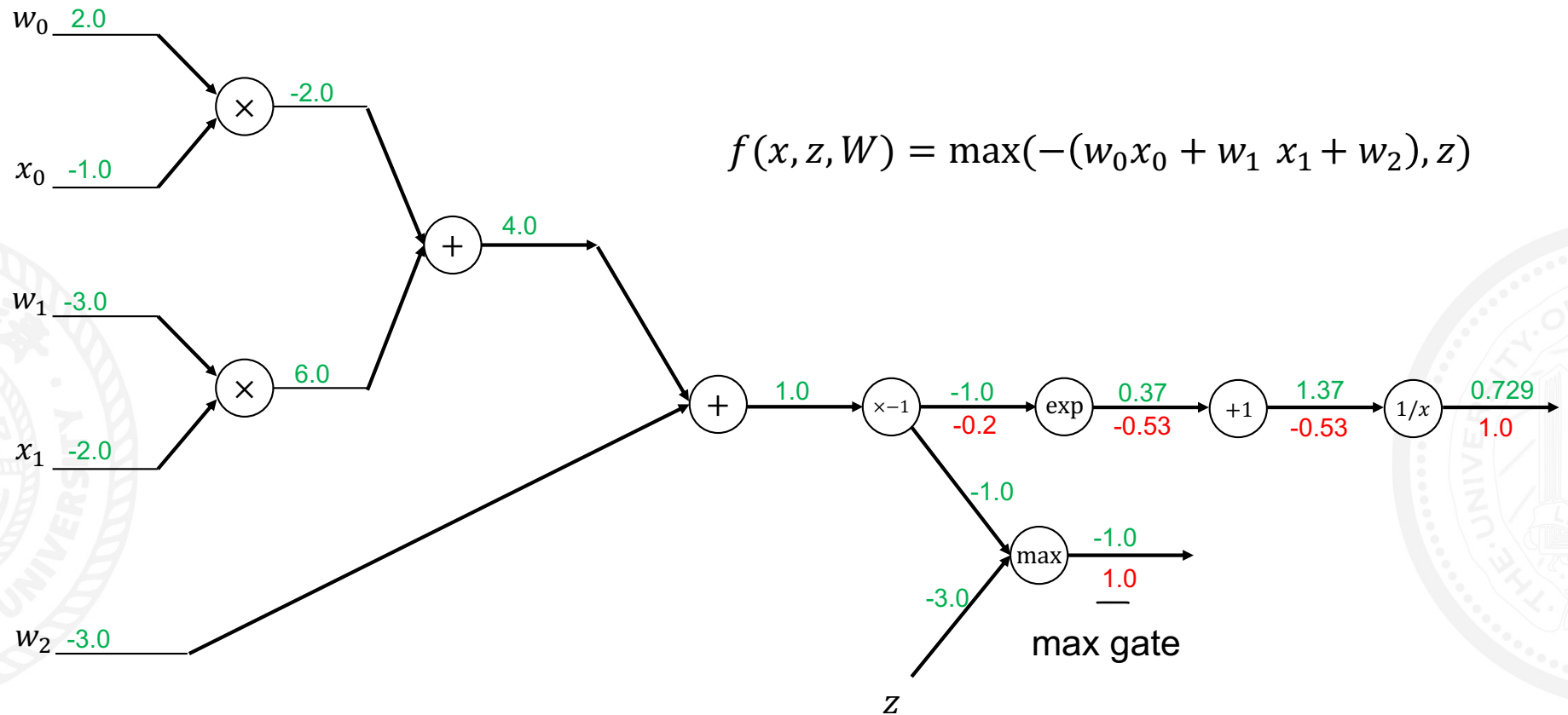
Backpropagation



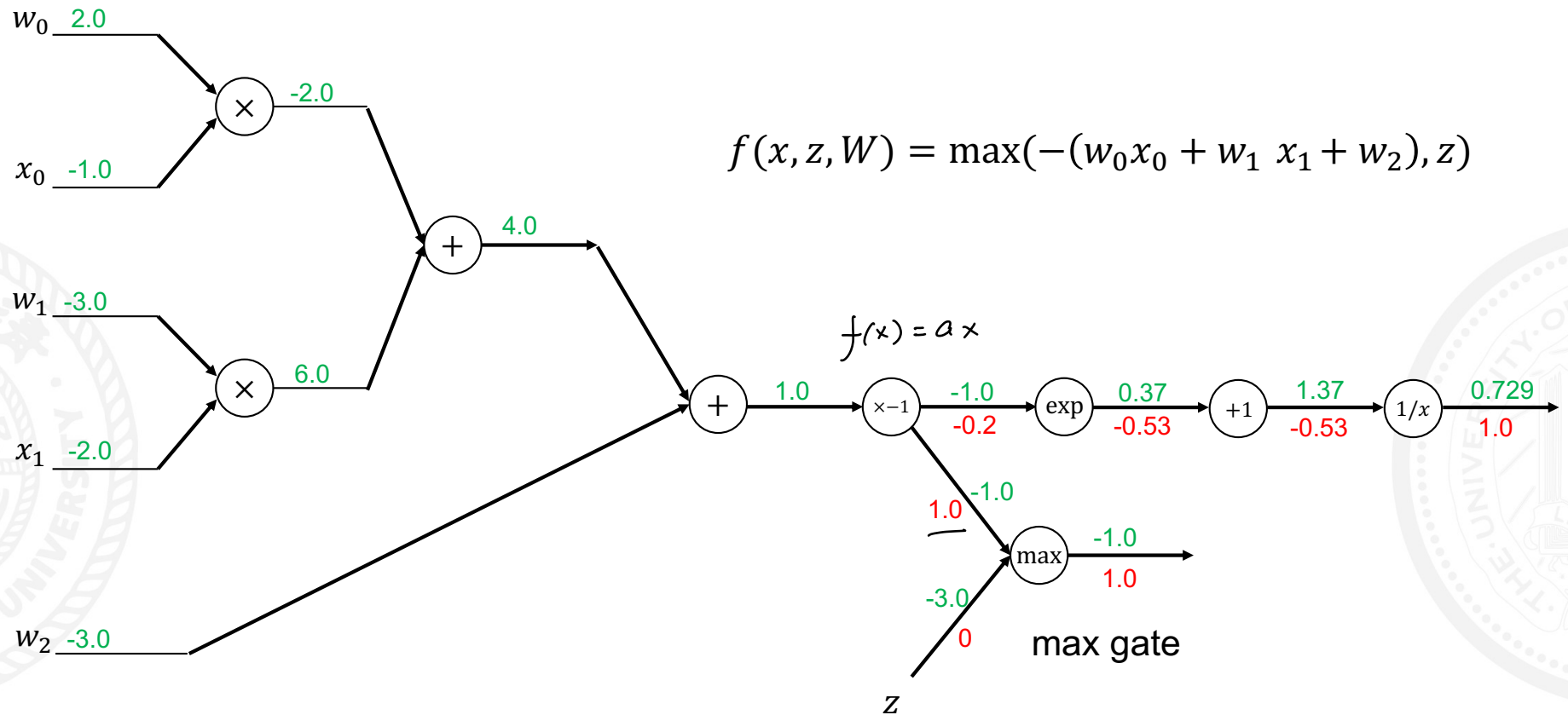
Backpropagation



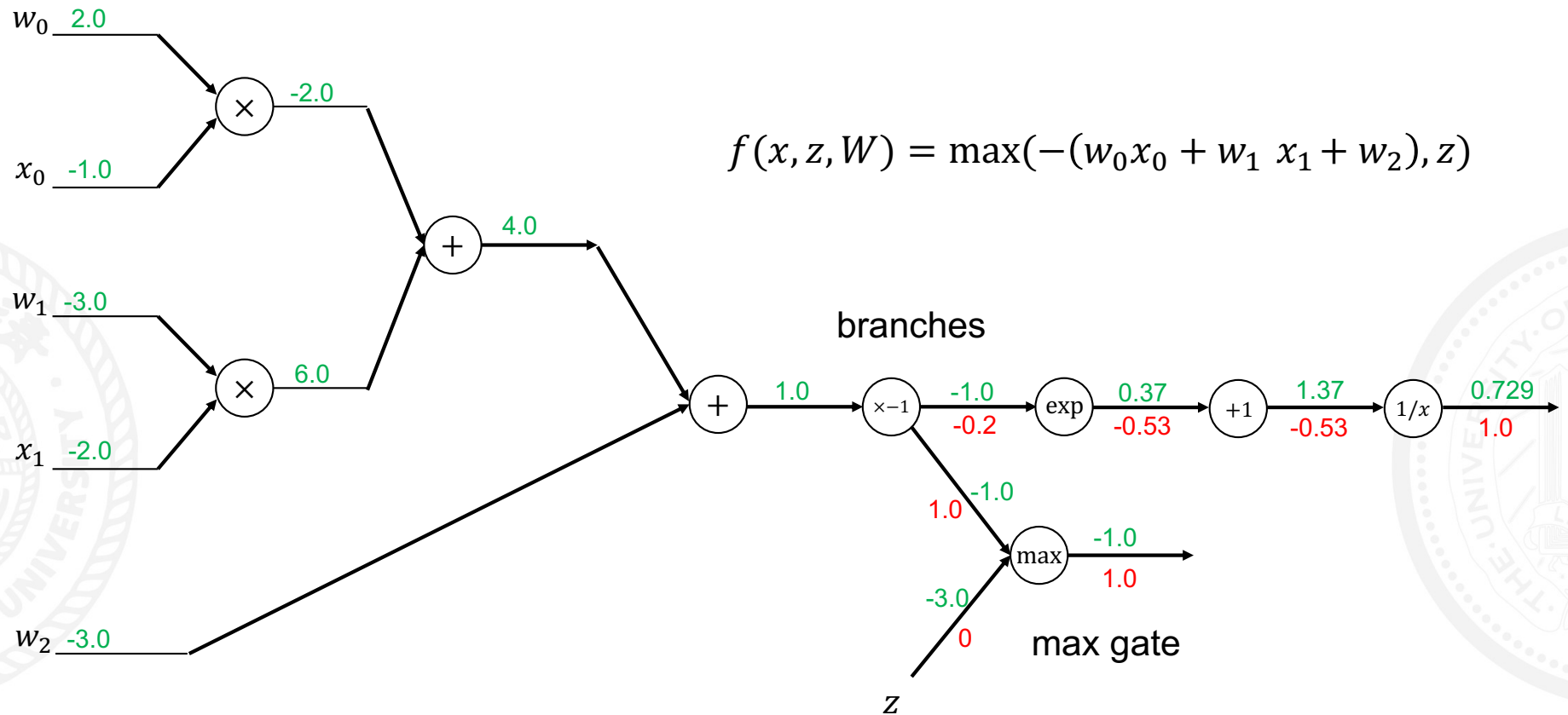
Backpropagation



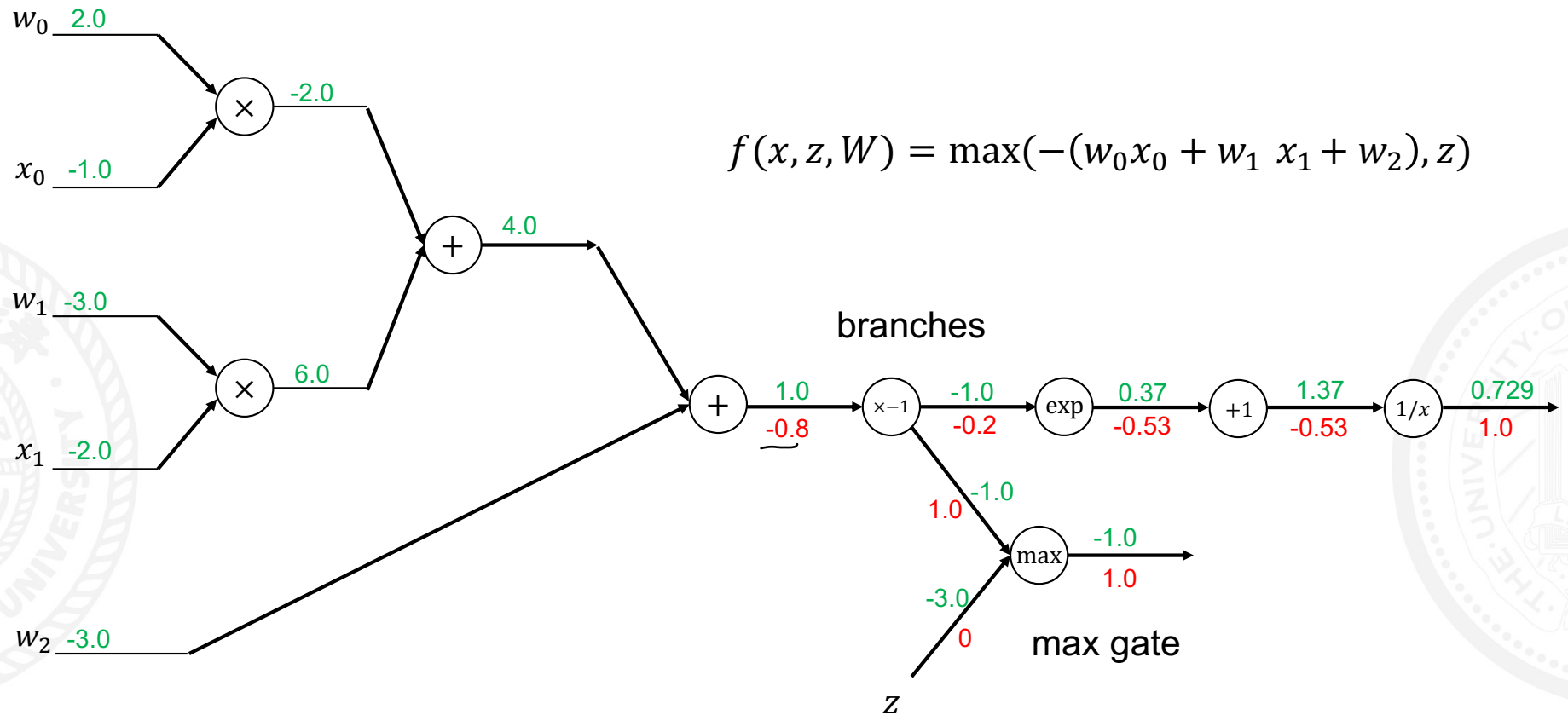
Backpropagation



Backpropagation



Backpropagation



Backpropagation

Vectorized example

$$f(\mathbf{x}, \mathbf{W}) = \|\mathbf{W} \cdot \mathbf{x}\|^2$$

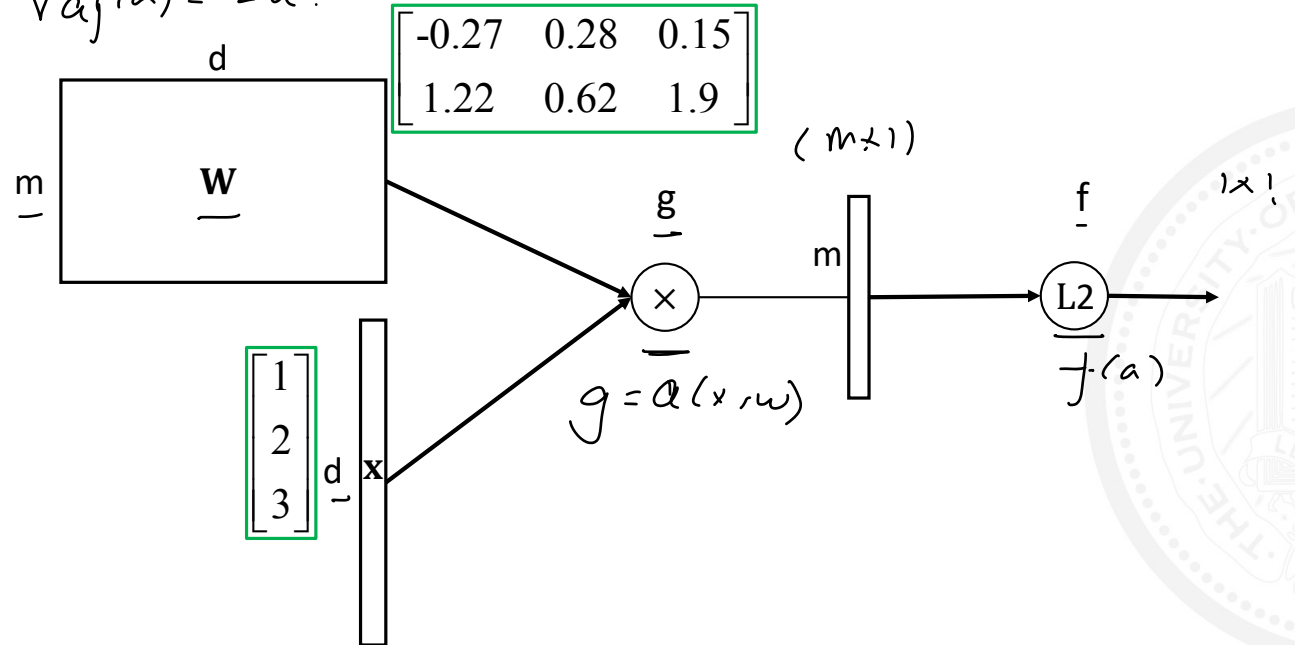
$$\textcircled{1} \quad \underline{a}(x, w) = \underline{x} w$$

$$\textcircled{2} \quad \underline{f}(a) = \|\underline{a}\|^2 = \underline{a}^T \underline{a}$$

$$\nabla_{\underline{a}} f(a) = 2 \underline{a}$$

$$\nabla_w \underline{a}(x, w) = \underline{x}^T$$

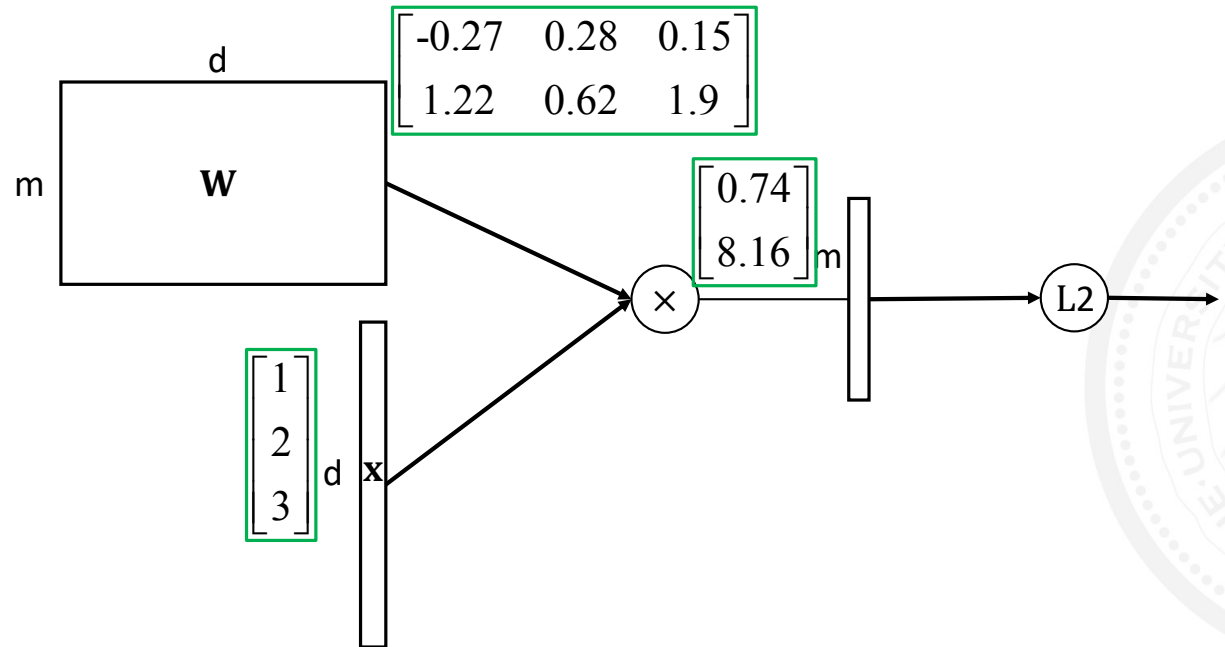
$$\nabla_x \underline{a}(x, w) = \underline{w}$$



Backpropagation

Vectorized example

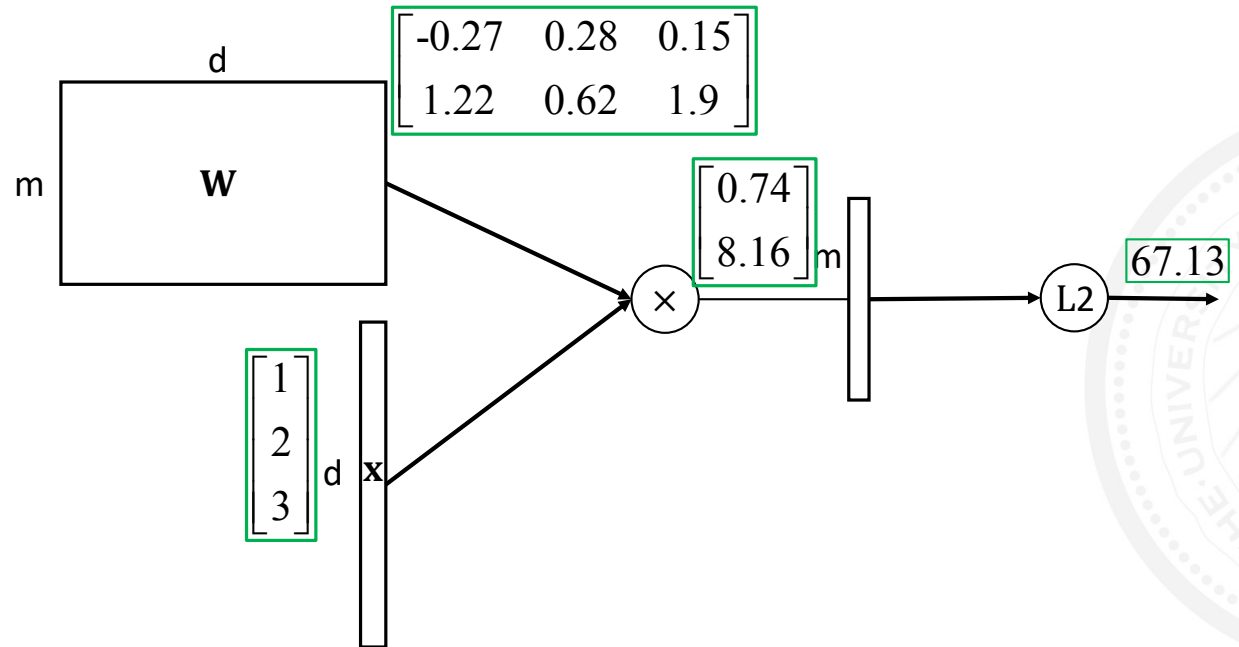
$$f(\mathbf{x}, \mathbf{W}) = \|\mathbf{W} \cdot \mathbf{x}\|^2$$



Backpropagation

Vectorized example

$$f(\mathbf{x}, \mathbf{W}) = \|\mathbf{W} \cdot \mathbf{x}\|^2$$



Backpropagation

Vectorized example

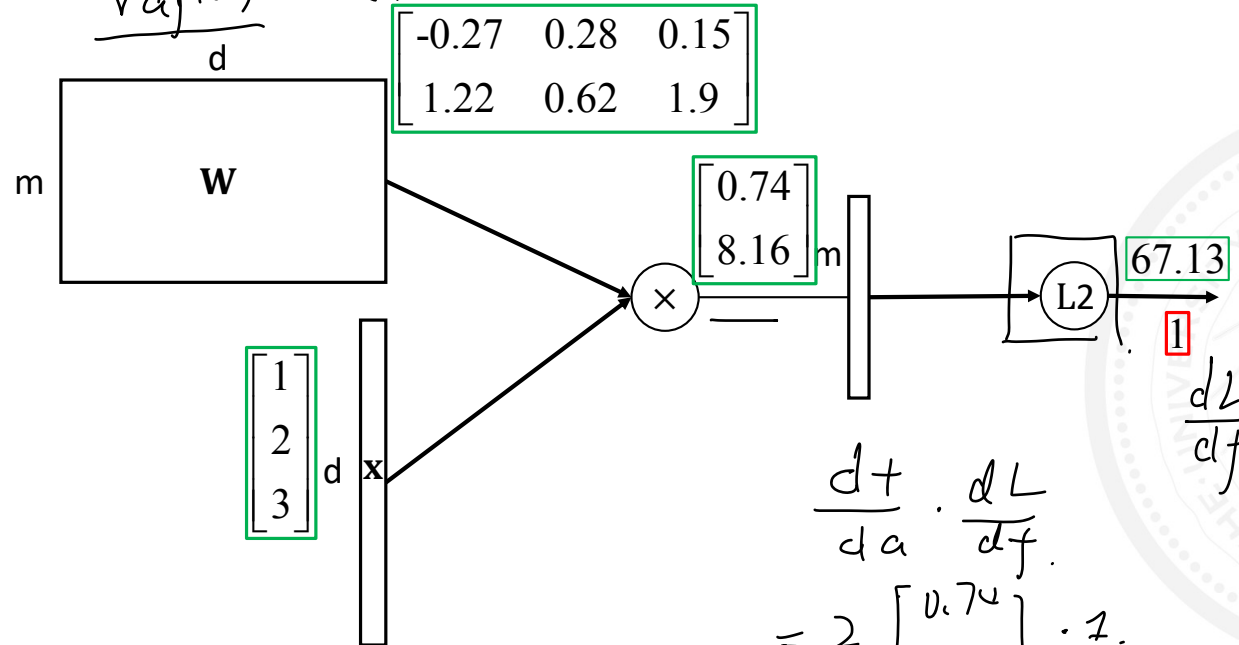
$$f(\mathbf{x}, \mathbf{W}) = \|\mathbf{W} \cdot \mathbf{x}\|^2$$

$$\textcircled{1} \quad \underline{a}(\mathbf{x}, \mathbf{w}) = \mathbf{x} \mathbf{w} \quad \nabla_{\mathbf{w}} a(\mathbf{x}, \mathbf{w}) = \mathbf{x}^T$$

$$\nabla_{\mathbf{x}} a(\mathbf{x}, \mathbf{w}) = \mathbf{w}$$

$$\textcircled{2} \quad \underline{f}(\mathbf{a}) = \|\mathbf{a}\|^2 = \mathbf{a}^T \mathbf{a}$$

$$\underline{\nabla_{\mathbf{a}} f(\mathbf{a})} = 2 \mathbf{a}$$



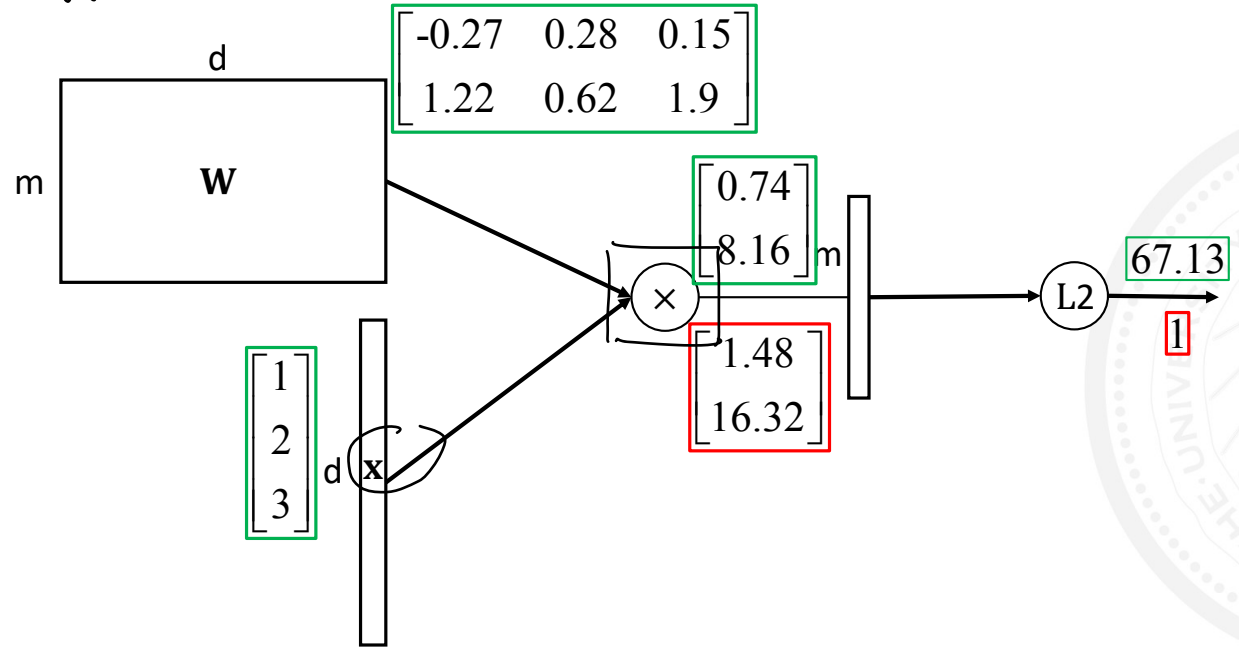
Backpropagation

Vectorized example

$$f(\mathbf{x}, \mathbf{W}) = \|\mathbf{W} \cdot \mathbf{x}\|^2$$

$a(\mathbf{W}, \mathbf{x}) = \mathbf{W} \mathbf{x}$

$$\begin{aligned} \frac{df}{d\mathbf{w}} &= \left(\frac{df}{da} \right) \left(\frac{da}{d\mathbf{w}} \right) \\ &= \begin{pmatrix} 1.48 \\ 16.32 \end{pmatrix} \mathbf{x}^T \\ &= \begin{pmatrix} 1.48 \\ 16.32 \end{pmatrix} (1 \ 2 \ 3) \end{aligned}$$



Backpropagation

Vectorized example

$$f(\mathbf{x}, \mathbf{W}) = \|\mathbf{W} \cdot \mathbf{x}\|^2$$

